Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (R) (FT/WP) Examinations April 2026 (2024 Scheme)

Course Code: GCMAT401

Course Name: MATHEMATICS FOR PHYSICAL SCIENCE-4

Max. Marks: 60 Duration: 2 hours 30 minutes

Normal distribution table and t distribution table are allowed in the examination hall

PART A

(Answer all questions. Each question carries 3 marks)

		CO	Marks
1	A chemical supply company currently has in stock 100 lb of a certain chemical, which it sells to customers in 5 lb batches. Let X be the number of batches ordered by a randomly chosen customer, and suppose that X has probability mass function (PMF) as follows. x: 1, 2, 3, 4 $p(x)$: 0.2, 0.4, 0.3, 0.1 Find the variance of X.	CO1	(3)
2	The mean and variance of a binomial random variable X are 6 and 3 respectively. Write the PMF of X.	CO1	(3)
3	In a city the daily consumption of milk in excess of 20,000gallons is approximately exponentially distributed with mean 3000 gallons. The city has a daily stock of 35,000 gallons. What is the probability that the stock is insufficient in a randomly selected day.	CO2	(3)
4	Let X be a normal random variable with mean 5 and variance 9. Find $P(\|X-2\|<4)$.	CO2	(3)
5	The population standard deviation for the height of student basketball players is 3 inches. If we want to be 95% confident that the sample mean height is within one inch of the true population mean height, how many randomly selected players must be surveyed?	CO3	(3)
6	A random sample of size 100 is drawn from a population with known standard deviation, $\sigma = 11.3$. The sample mean is 105.2. Construct a 95% confidence interval for the population mean.	CO3	(3)
7	Using Newton-Raphson method, find the square root of 3. Perform two iterations.	CO4	(3)

8		Use Euler method to solve the initial value problem, $\frac{dy}{dx} = x + xy + y$ with $y(0) = 1$.Compute the value of y at $x = 0.15$, by taking step size $h = 0.05$.	CO4	(3)

PART B

(Answer any one full question from each module, each question carries 9 marks)

Module -1

9	a)	A machine operates in two possible states represented by the random variable X .The states are defined as: $X = 0$ (Normal condition), $X = 1$ (Faulty condition). The probabilities of these states are: $P(X = 0) = 0.6$, $P(X = 1) = 0.4$. A sensor monitors the machine and produces an output represented by the random variable Y, where $Y = 0$ (Reports normal) and $Y = 1$ (Reports fault). Due to sensor inaccuracies, $P(Y = 1 \mid X = 0) = 0.2$ (false alarm) and $P(Y = 0 \mid X = 1) = 0.1$ (missed detection). (i) Find the joint probability distribution of X and Y. (ii) Are X and Y independent?	CO1	5
	b)	The possible values of a random variable X are 0,1,2,3. Given that, $P(X = 1) = 2P(X = 0)$; $P(X = 2) = 3P(X = 1)$; and $P(X = 3) = 2P(X = 2)$. Find (i) The probability mass function (PMF) of X (ii) The cumulative distribution function (CDF) of X	CO1	4
10	a)	The discrete random variables (X, Y) has joint PMF: $P(X = x, Y = y) = k(x + y)$: $x = 0,1,2$; $y = 0,1$. (i) Find the value of k (ii) Write the marginal distributions of X and Y (iii) Find $E(X), E(Y)$.	CO1	5
	b)	A concrete manufacturing company produces blocks, of which 2% are defective. These blocks are packed in batches of 150. (i) Find the probability that a randomly selected batch contains at most 3 defective blocks. (ii) If 800 such batches are produced, estimate the number of batches that are likely to contain more than 3 defective blocks.	CO1	4

Module -2

11	a)	The daily production of a machine follows a normal distribution with mean 250 units and standard deviation 25 units. Out of 1000 working days, estimate the number of days on which the machine output will be; (i) less than 200 units (ii) greater than 225 units	CO2	5
	b)	Let X be a random variable uniformly distributed over the interval $[-4,4]$. Define another random variable Y by $Y = X^2$. Show that	CO2	4

$E(XY) = E(X)E(Y)$ even though X and Y are dependent.

12	a)	The operating temperature of an engine is normally distributed. It is observed that 10% of the time, the temperature is below $70°C$ and 5% of the time, the temperature exceeds $110°C$. Find the percentage of time the engine temperature lies between $70°C$ and $90°C$.	CO2	5
	b)	The joint probability density function of two random variables X and Y is given by, $f(x,y) = \frac{1}{36}xy$; $0 < x < 3$, $0 < y < 4$. Find the probability that the sum of X and Y is less than 3.	CO2	4

Module -3

13	a)	A manufacturer claims that the mean lifetime of its electric motors is 1000 hours. To verify this claim, a researcher collects two independent samples: (i) Factory A A random sample of size 100 gives a sample mean lifetime of 980 hours and a sample standard deviation of 120 hours. (ii) Factory B A random sample of size 12 gives a sample mean lifetime of 950 hours and a sample standard deviation of 150 hours. Assume that the lifetimes from Factory B are approximately normally distributed. At the 5 % significance level, test in each case whether the true mean lifetime differs from 1000 hours.	CO3	5
	b)	A mobile application is tested on 250 users, out of which 180 report that they find it useful. Test, at the 5% level of significance, whether the proportion of users who find the application useful is 70%.	CO3	4
14	a)	A random sample of size 16 with sample mean 53, is taken from a normal population. The sum of squares of deviation from the sample mean is given as 135.Obtain a 95 %confidence limits for the mean of the population. Can this sample be regarded as taken from a population having mean 56?	CO3	5
	b)	A manufacturer claims that 85 %of its electric bulbs are defect-free. A quality inspector tests a sample of 12 bulbs and finds that only 8 are defect-free. At the 5 % significance level, test whether the true proportion of defect-free bulbs is less than 0.85 .	CO3	4

Module -4

15	a)	Solve the system using Gauss-Jordan elimination with partial pivoting: $0.005x + y - z = 2$; $x + 4y + 3z = 7$; $1.995x + 7y + 7z = 12$.	CO4	5
	b)	Using Second order Runge-Kutta method, solve $\frac{dy}{dx} = 1 + y - x^2$, with $y(0) = \frac{1}{2}$; for $0 \le x \le 0.4$ by taking step size $h = 0.2$.	CO4	4

16 a) Solve the Laplace equation $\nabla^2 u = 0$ with boundary conditions $u(0, y) = 100;\ u(2, y) = u(x, 0) = u(x, 2) = 0$.Compute the interior nodes with $0 < x < 2\ ;\ 0 < y < 2$ with mesh size $h = 0.5$. Perform two iterations only. CO4 5

b) Use the method of least squares, to fit a straight line of the form $y = ax + b$ to the following data. CO4 4

x	1	2	3	4
y	2	3	5	7

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