A 0100MAT101052405 Pages: 3

Reg No.:______________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1 (S,FE) S2 (S,FE) Degree Examination December 2025 (2019 Scheme)

Course Code: MAT101
Course Name: LINEAR ALGEBRA AND CALCULUS
(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1 Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & -3 & -4 \\ -1 & -4 & 9 & 18 \end{bmatrix}$ by reducing to echelon form. (3)

2 If one eigenvalue of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is 5, find the other eigenvalues without finding the characteristic equation. (3)

3 If f (x, y) = $x^3y + e^{xy^2}$, find $\frac{\partial^2 f}{\partial x \partial y}$. (3)

4 Find the critical points of the function f (x, y) = $2xy - x^3 - y^2$. (3)

5 Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. (3)

6 Find the area of the region enclosed by $y = x^2$ and $y = x$. (3)

7 Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges. If so, find the sum. (3)

8 Test the convergence of $\sum_{k=1}^{\infty} \frac{99^k}{k!}$. (3)

9 Determine the Taylor series expansion of f(x) = sinx at x = $\frac{\pi}{4}$. (3)

10 Express f(x) = x as a half range sine series in $0 < x < 2$. (3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following system of equations using Gauss elimination method. (7)

$$2x + y - z = 4$$
$$x - y + 2z = -2$$
$$x - 2y + z = -2.$$

b Find the eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$. (7)

12 a Test for consistency and solve (7)

$$2x + z = 3$$
$$x - y - z = 1$$
$$3x - y = 4\,.$$

b Diagonalize $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. (7)

MODULE 2

13 a Let f be a differentiable function of three variables and suppose that $w = f(x-y, y-z, z-x)$, show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (7)

b Let $f(x,y) = x^2 y$. The local linear approximation L to f at a point P is $L(x,y) = 4y - 4x + 8$. Determine the point P. (7)

14 a The length, width and height of a rectangular box are measured with an error of at most 5% .Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. (7)

b Find the absolute extrema of the function $f(x,y) = xy - 4x$ on R where R is the triangular region with vertices (0,0), (0,4) and (4,0). (7)

MODULE 3

15 a By reversing the order of integration, evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\, dxdy$. (7)

b Use triple integral to find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$. (7)

16 a Evaluate the double integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2)\, dxdy$ by converting to polar coordinates. (7)

b Evaluate $\iiint_G xyz dV$ where G is the solid in the first octant bounded by the parabolic cylinder $z = 3 - x^2$ and the planes $z = 0, y = x \, and \, y = 0$. (7)

MODULE 4

17 a Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$. (7)

b Examine the convergence of (i) $\sum_{k=1}^{\infty} \frac{k!10^k}{3^k}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{2k^2-1}$. (4+3)

18 a Determine whether the series $\sum_{k=1}^{\infty} (-1)^k \frac{k^4}{4^k}$ is absolutely convergent. (7)

b Examine the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{4k+1}$. (7)

MODULE 5

19 a Find the Fourier series for $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi < x \le 2\pi \end{cases}$. (7)

b Obtain the half range cosine series of $f(x) = x$ in the interval $0 \le x \le \pi$.Hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$. (7)

20 a If $f(x) = |x|$, expand $f(x)$ as a Fourier series in the interval $(-2,2)$. (7)

b Find the half range sine series for e^x in the interval $(0, l)$ (7)

****