Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S,FE) (FT/WP) (S2 PT) Examination December 2025/January 2026 (2019 Scheme)

Course Code: MAT206

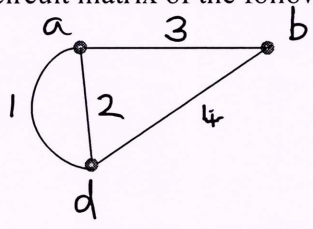
Course Name: GRAPH THEORY

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks)

		Marks
1	Prove that the sum of degree of all vertices in a graph is twice the number of edges	3
2	State Konigsberg bridge problem and draw its graph.	3
3	Define Euler graph and unicursal graph. Justify with an example that a connected graph is unicursal if it has exactly two vertices of odd degree.	3
4	Prove that in any digraph the sum of all indegrees is equal to the sum of all out degrees	3
5	Consider an electrical network containing 11 elements and 6 nodes. What is the minimum number of elements that must be removed so as to make the network circuitless?	3
6	Prove that the number of vertices in a binary tree is always odd.	3
7	Define fundamental circuit with an example.	3
8	What is a Planar graph? Give an example.	3
9	Write the circuit matrix of the following graph.	3



10		State Four color theorem.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

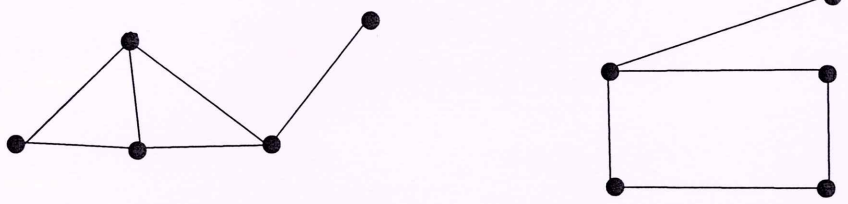
Module -1

11	a)	Define subgraph, edge disjoint subgraph and vertex disjoint subgraph with an example	7

b) Prove that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ $edges.$ 7

12 a) Determine whether the graphs are isomorphic. Justify your answer. 7

b



b) What is the largest possible number of vertices in a graph G with 35 edges and all vertices are of degree at least 3? Give the answer in detail. 7

Module -2

13 a) Prove that in a connected graph G with exactly 2k odd vertices, then there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph. 7

b) Define directed walk, directed path and directed circuit in a digraph. Give example 7

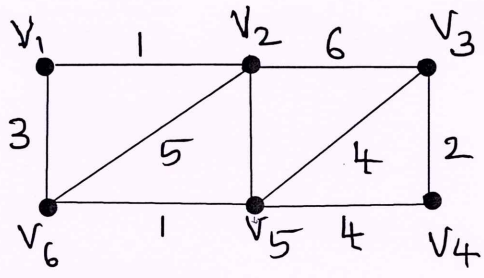
14 a) Prove that a connected graph is an Euler graph if and only if all vertices of the graph are of even degree. 7

b) Draw the digraph for the binary relation "is greater than" on X={3,4,5,7,8}. Also write its relation matrix. 7

Module -3

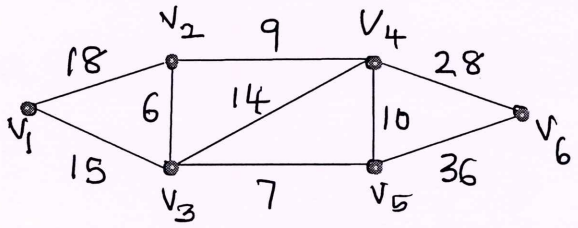
15 a) Prove that a tree with n vertices has (n-1) edges. 7

b) Using Prim's algorithm find the minimum spanning tree for the graph below. 7



16 a) Draw all unlabelled trees on 6 vertices. 7

b) Use Dijkstra's algorithm to find the shortest path between v_1 and v_5. 7



Module -4

17 a) Define edge connectivity. Prove that the edge connectivity of a graph cannot exceed the degree of the vertex with the smallest degree in G 7

b) Prove that the Kuratowski's second graph $K_{3,3}$ is non-planar. 7

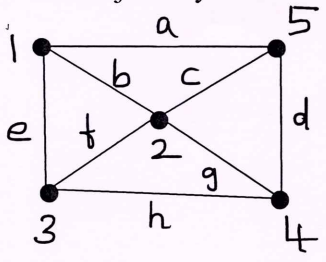
18 a) Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G. 7

b) Prove that a graph can be embedded on the surface of a sphere if and only if it can be embedded on a plane. 7

Module -5

19 a) Prove that every tree with two or more vertices is 2-chromatic 7

b) Write the incidence matrix and adjacency matrix of the following graph. 7



20 a) If A and B are the incidence matrix and circuit matrix respectively of a graph G whose columns are arranged using the same order of edges prove that $AB^T=BA^T=0$ (mod 2) 7

b) Prove that covering g of a graph is minimal if and only if g contains no path of length three or more. 7

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