Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1 (S,FE) S2 (S,FE) Degree Examination December 2025 (2019 Scheme)

Course Code: MAT 102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 SCHEME)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1 Find the velocity of a particle moving along the curve $\boldsymbol{r}(t) = e^t sint\boldsymbol{i} + e^t cost\boldsymbol{j} + t\,\boldsymbol{k}$ at $t = {}^{\pi}/_{2}$. (3)

2 If $\boldsymbol{r}(t) = x\,\boldsymbol{i} + \boldsymbol{y}\,\boldsymbol{j} + z\,\boldsymbol{k}$, *then find curl* $\boldsymbol{r}$ *and div* $\boldsymbol{r}$. (3)

3 Find the work done by the force field $\mathbf{F}(x, y) = (x^2 + y^2)\boldsymbol{i} + xy\boldsymbol{j}$ on a particle along a circular arc given by $x = cos\,t, y = sint, for\ 0 \le t \le \pi$. (3)

4 Using Stoke's theorem evaluate $\int_C \boldsymbol{F}.d\boldsymbol{r}$, where $\mathbf{F}(x, y, z) = (x + 2y)\boldsymbol{i} + (2x - 2yz)\boldsymbol{j} + (z^2 - y^2)\boldsymbol{k}$, where C is the path along $0 \le x \le 1, 0 \le y \le 3, and\ z = y$. (3)

5 Check whether $e^{-x}cosx$ *and* $e^{-x}sinx$ are linearly independent using Wronskian. (3)

6 Solve the differential equation $y''' - y = 0$ (3)

7 Find the Laplace transform of $t\,cos^2 t$. (3)

8 Evaluate the inverse Laplace transform of $\frac{e^{-s}}{s^2+1}$. (3)

9 Find the Fourier sine transform of $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ (3)

10 Write the Fourier cosine integral of $f(x) = \begin{cases} k, & 0 < x < c \\ 0, & x > c \end{cases}$ (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module I

11 a) Find the directional derivative of $f(x,y,z) = x^3z - yx^2 + z^2$ in the direction of $3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ at $(2,-1,1)$. Also find the unit vector in the direction along which maximum value of directional derivative occur. (7)

b) Evaluate $\int_C -ydx + xdy$ where C is $y^2 = 3x$ $from(3,3)$ to $(0,0)$. (7)

OR

12 a) Find the displacement and distance travelled by a particle with position vector **r**(t)= (1-3sint)$\boldsymbol{i}$ +3cost $\boldsymbol{j}$ over the interval $0 \leq t \leq \pi$. (7)

b) Prove that the vector $\boldsymbol{F} = (2xy + z^3)\boldsymbol{i} + x^2\boldsymbol{j} + 3xz^2\boldsymbol{k}$ is conservative vector field. Also find the scalar potential. (7)

Module II

13 a) Evaluate the surface integral $\iint_S xzdS$ where S is the part of the plane x+y+z=1 that lies in first octant . (7)

b) Using Green's theorem, evaluate $\int_C xcosy\ dx - ysinx\ dy$ where C is the square with vertices (0,0), $(0,\pi)$, $(\pi,0)$, (π,π). (7)

OR

14 a) Find the outward flux of the vector field $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{x}) = (x^2 - z)\boldsymbol{i} + (y - x)\boldsymbol{j} + (2z - y)\boldsymbol{k}$ across the surface $\boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is the cylindrical solid bounded by $x^2 + y^2 = a^2$ and z=0,z=1. (7)

b) Using Stoke's theorem, evaluate $\int_C y\ dx + z\ dy + xdz$ where C is the region bounded by $x^2 = y$ and $x = y$, oriented counter clock wise direction. (7)

Module III

15 a) Solve the initial value problem $y'' + 4y' + 5y = 0, y(0) = 2, y'(0) = -5$. (7)

b) Solve $y'' - 4y' + 3y = e^x cos2x$, by method of undetermined coefficients. (7)

OR

16 a) Solve $y'' + y = tan\ x$, by method of variation of parameters. (7)

b) Solve by method of undetermined coefficients, $y'' - 2y' + 5y = x^2$. (7)

Module IV

17 a) Using Convolution theorem, find the inverse of $\frac{s^2}{(s^2+1)(s^2+4)}$. (7)

b) Find the inverse Laplace transform of $\frac{e^{-2s}}{(s+1)^2}$ (7)

OR

18 a) Solve using Laplace transform, $y'' - 2y' + y = t^2e^t,\ y(0) = 1, y'(0) = 0.$ (7)

b) Find the Laplace transform of $f(t) = e^{-t}\ t\ cos3t.$ (7)

Module V

19 a) Find the Fourier integral of $e^{-a|x|}$. Hence Prove that $\int_0^\infty \frac{cos\ \omega x}{1+\omega^2}\ d\omega = \frac{\pi}{2}\ e^{-|x|}$. (7)

b) Find the Fourier sine transform of $f(x) = \begin{cases} x & : 0 < x < 1 \\ 3 - x & : 1 < x < 3 \\ 0 & : x > 3 \end{cases}$ (7)

OR

20 a) Find the Fourier cosine integral of $f(x) = \begin{cases} 1 - x^2 & : 0 < x < 1 \\ 0 & : otherwise \end{cases}$. (7)

Hence prove that $\int_0^\infty \frac{sin\ \omega - \omega cos\ \omega}{\omega^3} \cos\left(\frac{\omega}{2}\right) d\omega = \frac{3\pi}{16}$.

b) Find the Fourier transform of $f(x) = \begin{cases} k\ x & : a < x < b \\ 0 & : otherwise \end{cases}$. (7)

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