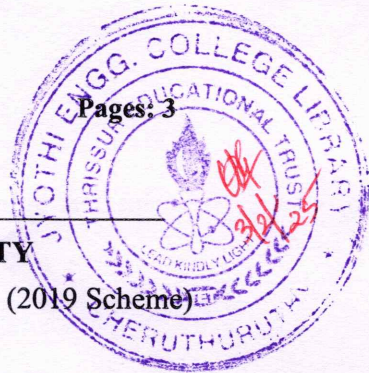


Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S, FE) (FT/WP) / S1 PT Examination December 2025 (2019 Scheme)



Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks

Marks

- 1 Find the differential equation of all spheres whose centers lie on z-axis (3)
- 2 Solve $\frac{\partial^2 z}{\partial y \partial x} = \frac{x}{y} + a$ (3)
- 3 Write down the possible solutions of one dimensional heat equation. (3)
- 4 Using d' Alemberts method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = a \sin^2 \pi x$ (3)
- 5 Determine 'a' so that $u = e^{-\pi x} \cos ay$ is harmonic. (3)
- 6 Prove that an analytic function of constant real part is constant. (3)
- 7 Find the Maclaurin series expansion of $f(z) = \frac{1}{1+z^2}$ (3)
- 8 Evaluate $\int_C \frac{1}{z^2+4} dz$ where C is $|z-2|=2$ (3)
- 9 Find the residue of $f(z) = \frac{\sin 2z}{z^2}$ at its pole. (3)
- 10 What type of singularity have the function $f(z) = e^{1/z}$ (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

- 11(a) Form the partial differential equation by eliminating arbitrary function from $lx + my + nz = \phi(x^2 + y^2 + z^2)$ (7)
- (b) Solve $p + q = pq$ (7)
- 12(a) Solve $(y+z)p - (z+x)q = x-y$ (7)
- (b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$. (7)

Module 2

- 13(a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3(\frac{\pi x}{l})$. If it is released from rest from this position, find the displacement $y(x, t)$. (7)
- (b) Find the temperature $u(x, t)$ in a homogeneous bar of heat conducting material of length l whose ends are kept at temperature 0°C and whose initial temperature is given by $\frac{ax(l-x)}{l^2}$. (7)
- 14(a) Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$, corresponding to the triangular initial deflection $f(x) = \begin{cases} \frac{2kx}{l} & \text{when } 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} & \text{when } \frac{l}{2} < x < l \end{cases}$ and initial velocity zero. (7)
- (b) A bar of 10cm long, with insulated sides has its ends A and B maintained at temperatures 50°C and 100°C respectively, until steady state conditions prevail. The temperature A is suddenly raised to 90°C and at the same time that at B is lowered to 60°C . Find the temperature distribution in the bar at time t . (7)

Module 3

- 15(a) Check whether the following functions are analytic or not. Justify your answer (i) $f(z) = iz\bar{z}$ (ii) $f(z) = e^{-z}$ (7)
- (b) Find the image of the lines $x = c_1$ and $x = c_2$ where c_1 and c_2 are non-zero constants under the transformation $w = z^2$. (7)
- 16(a) Find the analytic function whose imaginary part is $v(x, y) = e^{-2x} \sin 2y$. (7)
- (b) Find the image of $x = c = \text{const}$, $-\pi < y \leq \pi$ under $w = e^z$ (7)

Module 4

- 17(a) Use Cauchy's integral formula to evaluate $\int_C \frac{z}{z^2 + 4z + 3} dz$, C is the circle $|z + 1| = 2$. (7)
- (b) Evaluate $\int_C \text{Im}(z^2) dz$ counter clockwise around the boundary of the triangle with vertices $0, 1, i$ (7)
- 18(a) Find the Taylor series of $f(z) = \frac{1}{1+z}$ about $z = -i$ (7)
- (b) Evaluate using Cauchy's integral formula, $\int_C \frac{dz}{(z-2i)^2(z-\frac{i}{2})^2}$ where C is $|z| = 1$ (7)

Module 5

- 19(a) Find the Laurent's series expansion of $f(z) = \frac{1}{z(z-i)}$ about $z = i$ (4)
- (b) Show that $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = 2\pi$ (10)
- 20(a) Evaluate using Cauchy's residue theorem $\int_C \frac{z-23}{z^2-4z-5} dz$ where C is the circle (7)
- $|z - 2 - i| = 3.2$
- (b) Apply calculus of residues to evaluate $\int_0^\infty \frac{dx}{(1+x^2)^2}$ (7)