Reg No.:_______________ Name:___________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S, FE) (FT/WP) / S1 PT Examination December 2025 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Find the differential equation of all spheres whose centers lie on z-axis	(3)
2	Solve $\frac{\partial^2 z}{\partial y \partial x} = \frac{x}{y} + a$	(3)
3	Write down the possible solutions of one dimensional heat equation.	(3)
4	Using d' Alemberts method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = asin^2\pi x$	(3)
5	Determine $'a'$ so that $u = e^{-\pi x}cosay$ is harmonic.	(3)
6	Prove that an analytic function of constant real part is constant.	(3)
7	Find the Maclaurin series expansion of $f(z) = \frac{1}{1+z^2}$	(3)
8	Evaluate $\int_C \frac{1}{z^2+4} dz$ where C is $\|z-2\| = 2$	(3)
9	Find the residue of $f(z) = \frac{sin2z}{z^2}$ at its pole.	(3)
10	What type of singularity have the function $f(z) = e^{1/z}$	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a)	Form the partial differential equation by eliminating arbitrary function from $lx + my + nz = \phi(x^2 + y^2 + z^2)$	(7)
(b)	Solve $p + q = pq$	(7)
12(a)	Solve $(y + z)p - (z + x)q = x - y$	(7)
(b)	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ where $u(0, y) = 8e^{-3y}$.	(7)

Module 2

13(a)	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 sin^3(\frac{\pi x}{l})$. If it is released from rest from this position, find the displacement $y(x,t)$.	(7)
(b)	Find the temperature $u(x,t)$ in a homogeneous bar of heat conducting material of length l whose ends are kept at temperature $0^0 C$ and whose initial temperature is given by $\frac{ax(l-x)}{l^2}$	(7)
14(a)	Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$, corresponding to the triangular initial deflection $f(x) = \begin{cases} \frac{2kx}{l} & when\ 0 < x < \frac{l}{2} \\ \frac{2k(l-x)}{l} & when\ \frac{l}{2} < x < l \end{cases}$ and initial velocity zero.	(7)
(b)	A bar of 10cm long, with insulated sides has its ends A and B maintained at temperatures $50^0 C$ and $100^0 C$ respectively, until steady state conditions prevail. The temperature A is suddenly raised to $90^0 C$ and at the same time that at B is lowered to $60^0 C$. Find the temperature distribution in the bar at time *t*.	(7)

Module 3

15(a)	Check whether the following functions are analytic or not. Justify your answer (i) $f(z) = iz\bar{z}$ (ii) $f(z) = e^{-z}$	(7)
(b)	Find the image of the lines $x = c_1$ and $x = c_2$ where c_1 and c_2 are non-zero constants under the transformation $w = z^2$.	(7)
16(a)	Find the analytic function whose imaginary part is $v(x,y) = e^{-2x} sin2y$.	(7)
(b)	Find the image of $x = c = const,\ -\pi < y \leq \pi$ under $w = e^z$	(7)

Module 4

17(a)	Use Cauchy's integral formula to evaluate $\int_C \frac{z}{z^2+4z+3} dz$, C is the circle $\lvert z+1 \rvert = 2$.	(7)
(b)	Evaluate $\int_C Im(z^2) dz$ counter clockwise around the boundary of the triangle with vertices $0, 1, i$	(7)
18(a)	Find the Taylor series of $f(z) = \frac{1}{1+z}$ about $z = -i$	(7)
(b)	Evaluate using Cauchy's integral formula, $\int_C \frac{dz}{(z-2i)^2(z-\frac{i}{2})^2}$ where C is $\lvert z \rvert = 1$	(7)

Module 5

19(a) Find the Laurent's series expansion of $f(z) = \frac{1}{z(z-i)}$ about $z = i$ (4)

(b) Show that $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-cos\theta} = 2\pi$ (10)

20(a) Evaluate using Cauchy's residue theorem $\int_C \frac{z-23}{z^2-4z-5} dz$ where C is the circle $|z-2-i| = 3.2$ (7)

(b) Apply calculus of residues to evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$ (7)