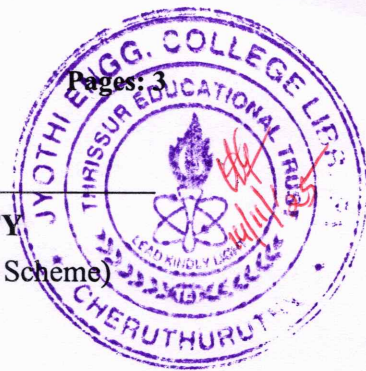


Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R) (FT/WP) Examination November 2025 (2024 Scheme)



Course Code: GYMAT301

Course Name: MATHEMATICS FOR PHYSICAL SCIENCE – 3

Max. Marks: 60

Duration: 2 hours 30 minutes

PART A

(Answer all questions. Each question carries 3 marks)

CO Marks

- | | | | |
|---|--|---|-----|
| 1 | State the conditions under which a function $f(x)$ has a Fourier integral representation and write the Fourier integral representation of the function under the conditions. | 1 | (3) |
| 2 | Find the Fourier sine transform of the function $f(x) = \begin{cases} 1; & \text{if } 0 < x < 1 \\ 0; & \text{if } x > 1 \end{cases}$ | 1 | (3) |
| 3 | Find $Re(f)$ and $Im(f)$ of the function $f(z)$ given by $f(z) = \frac{1}{1-z}$. | 2 | (3) |
| 4 | Sketch and shade the region in the complex plane give by $ z - 1 + i \leq \sqrt{2}$. | 2 | (3) |
| 5 | Evaluate $\oint_C \frac{2}{z-4i} dz$ where C is the circle $ z = \pi$ counter clockwise. | 3 | (3) |
| 6 | Evaluate $\int_C \sec^2 z dz$ where C is any path from $z = -\pi/4$ to $z = \pi/4$. | 3 | (3) |
| 7 | Find the location and order of zeros of $f(z) = \sin^2 z - 1$. | 4 | (3) |
| 8 | Find the residue of the pole $z = 0$ of the function $f(z) = z^{-5} \cos z$. | 4 | (3) |

PART B

(Answer any one full question from each module, each question carries 9 marks)

Module -1

- 9 a) Find the Fourier integral representation of the function $f(x)$ given by $f(x) = \begin{cases} 1, & \text{if } |x| < 2 \\ 0, & \text{if } |x| > 2 \end{cases}$. Find the function to which it converges. 1 5
- b) If $f(x)$ is continuous and absolutely integrable on the x-axis, $f'(x)$ is piecewise continuous on every finite interval and if $f(x) \rightarrow 0$ as $x \rightarrow \infty$, then prove that (i) $\mathcal{F}_c[f'(x)] = \omega \mathcal{F}_s[f(x)] - \sqrt{\frac{2}{\pi}} f(0)$,
(ii) $\mathcal{F}_s[f'(x)] = -\omega \mathcal{F}_c[f(x)]$. 1 4
- 10 a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{Otherwise} \end{cases}$. Hence evaluate $\int_0^\infty \frac{1 - \cos x}{x^2} dx$. 1 5
- b) Find the Fourier cosine integral of $f(x) = e^{-2x}, x > 0$. Hence evaluate $\int_0^\infty \frac{\cos \omega x}{4 + \omega^2} dx$. 1 4

Module -2

- 11 a) Check whether the function $u(x, y) = e^x \sin y$ is harmonic. If yes, find the harmonic conjugate $v(x, y)$ and the analytic function $f(z) = u + iv$. 2 5
- b) Find and sketch the image of the region $1 < |z| < 2, 0 < \text{Arg } z < \frac{\pi}{2}$ under the mapping $w = z^2$. 2 4
- 12 a) Define an analytic function in a domain D . Show that $f(z) = |z|^2$ is differentiable only at $z = 0$ and hence it is nowhere analytic. 2 5
- b) Determine the image of the straight line $x = 1$ under the mapping $w = \frac{1}{z}$. Sketch the image. 2 4

Module -3

- 13 a) Find $\int_C z e^{z^2} dz$ where C is the path through the axes from $z = 1$ to $z = i$. 3 5
- b) Evaluate $\int_C \frac{2z}{(z-1)(z-3)} dz$ where C is the circle given by: (i) $|z - 1| = 1$, (ii) $|z - 2| = 2$. 3 4
- 14 a) Evaluate $\oint_C \frac{3z-1}{z^2-z-2} dz$ where C is the circle given by (i) $|z - 1 - i| = 3$. 3 5

- b) Evaluate $\int_C \bar{z} dz$ where C is parametrized by $z(t) = 3t + it^2, -1 \leq t \leq 4$. 3 4

Module -4

- 15 a) Evaluate $\int_0^{2\pi} \frac{1}{5-2\sin\theta} d\theta$. 4 5
- b) Find the first three non-zero terms of the Taylor's series of the function $f(z) = \cos^2 z$ at the point $z = \pi$. 4 4
- 16 a) Find all the poles and nature of poles of the function $f(z) = \frac{z+1}{z^3-2z^2}$. 4 5
- By finding the residues at the pole, evaluate $\oint_C \frac{z+1}{z^3-2z^2} dz$ where C is the circle $|z-1| = 2$.
- b) Find the Laurent's series representation of the function $f(z) = \frac{2z+3}{z^2+z-6}$ in the region $2 \leq |z| \leq 3$. 4 4
