Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R) (FT/WP) Examination November 2025 (2024 Scheme)

Course Code: GAMAT301

Course Name: MATHEMATICS FOR INFORMATION SCIENCE-3

Max. Marks: 60 Duration: 2hours 30minutes

PART A

	(Answer all questions. Each question carries 3 marks)	CO	Marks
1	Find the probability distribution function of a Binomial distribution, given that the number of trials is 5 and the sum of its mean and variance is 1.8.	1	(3)
2	A discrete random variable X has the probability distribution $P(X = x) = \frac{k}{2^x}$; x = 0, 1, 2, 3, 4. Find (i) the value of k (ii) the probability that X takes an even value.	1	(3)
3	The cumulative distribution function of a continuous random variable X is defined by: $F(x) = \begin{cases} 0 & , x \le 2 \\ k(x-2) & , 2 < x < 6. \\ 1 & , x \ge 6 \end{cases}$ Find (i) the value of k. (ii) $P(X > 4)$	2	(3)
4	If X follows an exponential distribution with $P(X \le 1) = P(X > 1)$, find mean and variance of X.	2	(3)
5	Define and provide examples for the classification of random processes.	3	(3)
6	Let $X_1, X_2, \ldots, X_{10}$ be independent Poisson random variables with mean 1. Use the Markov inequality to get a bound on $P(X_1 + X_2 + \ldots + X_{10} \ge 15)$.	3	(3)
7	State Chapman-Kolmogorov theorem in homogeneous Markov Chain.	4	(3)
8	If the transition probability matrix of a Markov chain is $P = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix}$, find the steady state distribution of the chain.	4	(3)

PART B

(Answer any one full question from each module, each question carries 9 marks)

Module -1

9	a)	Derive the mean and variance of the Poisson Distribution.	1	(4)
	b)	The joint probability mass function of two random variables X and Y is given by $P(x, y) = k(x + 2y)$; $x = 0, 1, 2$; $y = 0, 1, 2, 3$. Find (i) the value of k. (ii) marginal density functions of X and Y. (iii) $P(X+Y \leq 3)$	1	(5)
10	a)	In an examination, a candidate is required to answer 15 multiple-choice questions, each having four possible options. He knows the correct answers to 10 of these questions, and for the remaining 5 questions, he selects an answer at random. (i) What is the probability that he answers 13 or more questions correctly? (ii) What is the mean and variance of the total number of correct answers he gives?	1	(5)
	b)	It is known that 2% of the bolts produced by a company are defective. The bolts are supplied in boxes of 200 bolts. What is the probability that a randomly chosen box contains no more than 5 defective bolts? In a consignment of 1000 such boxes, how many can be expected to have more than 5 defective bolts?	1	(4)

Module -2

11	a)	X is uniformly distributed with mean 1 and variance $\frac{4}{3}$. If three independent observations of X are made, what is the probability that all of them are negative?	2	(5)
	b)	The life times of tube light bulbs produced by a company are normally distributed with mean 1000 hrs and standard deviation 100 hrs. Is this company correct when it claims that 95% of its light bulbs last at least 900 hrs?	2	(4)
12	a)	The joint probability distribution function of two continuous random variable X and Y is: $f(x, y) = \begin{cases} 8xy, & 0 < y < x < 1 \\ 0, & otherwise \end{cases}$ (i) Find $P(X+Y < 1)$ (ii) Check whether X and Y are independent	2	(5)
	b)	Suppose a new machine is put into operation at time zero. Its life time is an exponential random variable with mean life 12 hrs. (i) What is the probability that the machine will work continuously for one day?	2	(4)

(ii) Suppose the machine has not failed by the end of the first day, what is the probability that it will work for the whole of the next day?

Module -3

13	a)	In a game involving repeated throws of a fair die, a person receives ₹3 if the number obtained is greater than or equal to 3, and loses ₹3 otherwise. Using the Central Limit Theorem, find the probability that after 25 throws, the person's total earnings exceed ₹25.	3	(5)
	b)	A radioactive source emits particles at a rate of 5 per minute in accordance with a Poisson process. Each particle emitted has a probability of 0.6 of being recorded by a device. (i) Find the probability that 10 particles are recorded in a four-minute period. (ii) On the average how many particles go unrecorded in a one-hour period? (iii) What is the expected time until 10^{th} the particle is recorded?	3	(4)
14	a)	Suppose the number of items produced in a factory during a week is a random variable with a mean of 500 and a variance of 100. What is the probability that the production in a given week will be between 400 and 600?	3	(4)
	b)	The number of failures occurring in a computer network follows a Poisson process. On average, one failure occurs every four hours. Find the probability of: (i) At most one failure occurring in the first 8 hours. (ii) At most one failure occurring in the first 8 hours and at least two failures occurring in the next 8 hours.	3	(5)

Module -4

15	a)	Three boys — A, B, and C — are throwing a ball to one another. A always throws the ball to B, and B always throws the ball to C, but C is equally likely to throw the ball to either A or B. (i) Considering this process as a Markov chain, construct the transition probability matrix. (ii) If the ball is initially with C, find the probability that it is with B after two passes. (iii) Determine the steady-state (long-run) probabilities of finding the ball with each of them.	4	(5)

b) Consider a Markov chain with state space S = {1, 2, 3, 4, 5} and the following transition matrix: 4 (4)

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.5 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 \\ 0 & 0 & 0 & 1.0 & 0 \end{bmatrix}$$

(i) Identify the communicating classes of the Markov chain.

(ii) Classify each communicating class as **recurrent** or **transient**.

(iii) Determine whether the Markov chain is **irreducible** and **aperiodic**.

16 a) A Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ is defined on the state space $S = \{1, 2, 3\}$ with the initial distribution 4 (4)

$P(X_0 = i) = \frac{1}{3}, i = 1, 2, 3;$ and TPM $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$.

Find **(i)** $P(X_2 = 3)$ **(ii)** $P(X_1 = 1, X_2 = 2, X_3 = 3)$ **(iii)** $P(X_2 = 2, X_3 = 3 \mid X_1 = 1)$

b) A gambler starts with a capital of ₹2 and plays a fair coin-toss betting game. In each round, he wins ₹1 if a head appears and loses ₹1 if a tail appears. The gambler stops playing as soon as his capital either increases to ₹4 or reduces to ₹0. Model this situation as a Markov chain and determine the transition probability matrix. 4 (5)

Find

(i) The probability that the gambler loses all his money at the end of two plays.

(ii) The probability that game ends exactly after two rounds.

(iii) The probability that the gambler proceeds to the third round.

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