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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S1 (S,FE) S2 (S,FE) Degree Examination May 2025 (2019 Scheme)

Course Code: MAT102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019-Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks.

1	Find the gradient of $\varphi(x,y,z) = xy + yz + zx$ at the point $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$.	(3)
2	Evaluate the line integral $\int_C (1 + xy^2)\, dS$ where C is $\vec{r}(t) = t\vec{i} + 2t\vec{j}$, $0 \le t \le 1$.	(3)
3	Determine the sources and sinks of the vector field $\vec{f}(x,y) = xy\vec{i} - 2xy\vec{j} + y^2\vec{k}$.	(3)
4	Evaluate the surface integral $\iint_\sigma (x - y - z)\, dS$ where σ is the part of the plane $x + y = 1$ that lies in the first octant between $z = 0$ and $z = 2$.	(3)
5	Solve the initial value problem, $y'' - 9y = 0,\ y(0) = 1,\ y'(0) = 1$.	(3)
6	Find the Wronskian corresponding to the solution of differential equation $y'' + 25y = 0$	(3)
7	Find the Laplace Transform of $\cos 2t \cos t$	(3)
8	Find the Laplace Transform of $f(t) = t\,\delta(t-1)$	(3)
9	Find the Fourier Sine Integral of $f(x) = \begin{cases} a - x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$.	(3)
10	Find the Fourier Cosine transform of $f(x) = 1,\ 0 < x < 1$	(3)

PART B

Answer one full question from each module, each question carries 14 marks

Module-I

11	a)	Find the parametric equation of the tangent line to the curve $\vec{r}(t) = (3t-1)\vec{i} + \sqrt{(3t+4)}\,\vec{j}$ at (-1,2)	(7)
	b)	Find the work done by the force $\vec{f}$ on a particle that moves along the curve C if $\vec{f} = xy\vec{i} + x^3\vec{j}$ and C is $x = y^2$ from (0,0) to (1,1)	(7)

12 a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$ prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ (7)

b) Show that the vector field $\vec{f}(x,y) = 3x^2 e^y \vec{i} + x^3 e^y \vec{j}$ is conservative and find ϕ such that $\vec{f} = \nabla\phi$. Hence evaluate $\int_{(0,0)}^{(3,2)} 3x^2 e^y\, dx + x^3 e^y\, dy$. (7)

Module-II

13 a) Use the Divergence theorem to find outward flux of $\vec{f}(x,y,z) = (x^2 + y)\vec{i} + xy\vec{j} - (2xz + y)\vec{k}$ where σ is the surface of the tetrahedron bounded by $x + y + z = 1$ and the coordinate planes. (7)

b) Use Green's theorem to evaluate $\int_C (xy + y^2)\, dx + x^2\, dy$ where C is the boundary of the area common to the curves $y = x^2$ *and* $y = x$. (7)

14 a) Find the flux of the vector field $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ across the portion of the surface $z = 1 - x^2 - y^2$ that lies above the XY-plane oriented by unit normal with positive components. (7)

b) Use Stokes theorem to evaluate $\oint_C \vec{f}.dr$ where $\vec{f}(x,y) = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ and C is the rectangle in the XY-plane bounded by the lines $x = 0, y = 0, x = a, y = b$ (7)

Module-III

15 a) Using the method of undetermined coefficients, solve $y'' - 2y' + 5y = x^2$ (7)

b) By the method of variation of parameter, solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ (7)

16 a) Solve the initial value problem $x^2 y'' + 3xy' + y = 0,\ y(1) = 0,\ y'(1) = 1$ (7)

b) Using the method of undetermined coefficients, solve $y'' - 4y' + 3y = \sin 3x$ (7)

Module-IV

17 a) Using Laplace Transform solve $y'' + 9y = 10e^{-t}$, $y(0) = 0,\ y'(0) = 0$ (7)

b) Using convolution theorem, find the inverse Laplace transform of $\frac{a}{s^2(s^2 - a^2)}$. (7)

18 a) Express the function $f(t) = \begin{cases} 2, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \\ \sin t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace Transform. (7)

b) Find the inverse Laplace Transform of $\dfrac{4s+5}{(s+1)^2(s-2)}$ (7)

Module-V

19 a) Using Fourier integral representation show that

$$\int_0^\infty \frac{\cos xw + w\sin xw}{1+w^2}\,dw = \begin{cases} 0 & \text{if} \quad x<0 \\ \frac{\pi}{2} & \text{if} \quad x=0 \\ \pi e^{-x} & \text{if} \quad x>0 \end{cases}$$ (7)

b) Find the Fourier Transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$ (7)

20 a) Represent $f(x) = \begin{cases} e^x, & 0<x<1 \\ 0, & x>1 \end{cases}$ as a Fourier Cosine integral (7)

b) Find the Fourier Sine Transform of $f(x) = \begin{cases} x, & \text{if} \quad 0<x<1 \\ 2-x & \text{if} \quad 1<x<2 \\ 0 & \text{if} \quad x\geq 2 \end{cases}$ (7)

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