Reg No.:_______________ Name:_______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S,FE) (FT/WP) (S1 PT) Examination May 2025 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential Equations and Complex Analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Form the partial differential equation by eliminating arbitrary constant from the relation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.	(3)
2	Solve $py + xq - xz = 0$	(3)
3	Write any three assumptions used in the derivation of one-dimensional wave equation.	(3)
4	Find the steady state temperature distribution in a rod of 10cm having its ends at $20^{\circ}c$ and $40^{\circ}c$ respectively.	(3)
5	Show that an analytic function $f(z) = u + iv$ is constant if its real part is constant	(3)
6	Test the continuity of $f(z)$ at the point $z = 0$, where f(z)=$\begin{cases} \frac{z}{\lvert z\rvert}, z \neq 0 \\ 0, \ z = 0 \end{cases}$	(3)
7	Find the value of the integral $\int_C z^2 \, dz$ where C is the part of the unit circle from –i to i in the right half plane.	(3)
8	Evaluate $\int_c \tan z \, dz$, where C is the circle $\lvert z\rvert = 1$.	(3)
9	Find the Laurent series expansion of $\frac{1}{1+z}$ about z=0	(3)
10	Find the singularities and nature of singularities of the function $f(z) = \frac{1-\sin z}{z}$	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Form the partial differential equation by eliminating arbitrary function from the relation $f(x^2+y^2, z-xy)=0$. (7)

(b) Solve the PDE $x(y-z)p+y(z-x)q=z(x-y)$

(7)

12(a) Using Charpit's method, solve $z=pq$. (7)

(b) Solve by Method of Separation of variable $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u, u(x,0)=6e^{-3x}$.

(7)

Module 2

13(a) A tightly stretched string with fixed ends at $x=0$ and $x=l$ is given a shape defined $f(x)=\lambda x(l-x)\ at\ t=0$, and then released. Find the displacement of the string at any distance x from one end at any time t. (7)

(b) Derive all possible solutions of one-dimensional heat equation.

(7)

14(a) Derive One Dimensional wave equation. (7)

(b) A homogeneous rod of heat conducting material of length 100cm has its ends kept at 0° temperature and the initial temperature is (7)

$$u(x,0)=\begin{cases} x, & \text{if } 0\le x\le 50 \\ 100-x, & \text{if } 50\le x\le 100 \end{cases}$$

Find the temperature $u(x,t)$ at any time t.

Module 3

15(a) Check whether $f(z)=log\ z$ is analytic or not. (7)

(b) Find the image of $1\le|z|\le 2, \frac{\pi}{4}\le\theta\le\frac{\pi}{2}$ under the mapping $w=z^2$.

(7)

16(a) Determine a so that the function $u = e^{2x} \cos ay$ is harmonic and find the conjugate. (7)

(b) Show that if $f(z) = u + iv$ is analytic, then it satisfies Cauchy-Riemann equation. (7)

Module 4

17(a) Evaluate $\int_C \frac{3z^2+z}{(z^2-9)(z-1)} dz$ where C is the circle $|z-1| = 1$. (7)

(b) Find the Taylor series expansion of $f(z) = \cos 2z$ about $z = \pi$. (7)

18(a) Evaluate $\int_C \bar{z}\, dz$ where C is the circle $|z| = 2$. (7)

(b) Evaluate $\int_C \frac{\sin \pi z}{(z-1)^4} dz$ where C is the circle $|z+1| = 3$. (7)

Module 5

19(a) Find Laurent series expansion $f(z) = \frac{1}{(z^2-3z+2)}$ valid in (7)

i. $1 \leq |z| \leq 2$

ii. $|z| \geq 2$

(b) Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$ (7)

20(a) Evaluate $\oint_C \frac{z-2}{z^2-4z-5} dz$, where C is circle $|z-2-i| = 4$. (7)

(b) Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^3}$ (7)

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