Reg No.:_______________ Name:___________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S,FE) (FT/WP) (S1 PT) Examination May 2025 (2019 Scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Show that $\neg(p\to q) \to \neg q$ is a tautology. Use Truth table.	(3)
2	Show that $R \vee M$, $\neg R \vee S$, $\neg M$, $\neg S$ cannot exist simultaneously. (without using truth table)	(3)
3	Determine the sum of all the coefficients in the expansion of $(x+y)^{11}$	(3)
4	A student is to answer 7 out of 10 questions in an examination. In how many ways can she make her selection if (i) there is no restriction (ii) she must answer the first two questions compulsorily.	(3)
5	Let A, B be sets with $\vert B \vert=3$. If there are 4096 relations from A to B, what is $\vert A \vert$? (Note: $\vert A \vert$ denotes number of elements in A.)	(3)
6	Show that the function f(x) = 2x + 3 where f: $R\to R$ is one-to-one.	(3)
7	Find the generating function for the sequence 0,0,0,6,-6,6,-6,6....	(3)
8	Find the sequence generated by the exponential generating function $e^x + x^2$	(3)
9	Show that the set of all n x n diagonal matrices are closed with respect to multiplication.	(3)
10	Prove that a cyclic monoid is always abelian.	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a)	Write the following argument in symbolic form and establish its validity. If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica, their attorney, will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.	(8)

(b) Negate and simplify the statement: $\forall x[p(x) \wedge \neg q(x)]$ (6)

12(a) Establish the following argument by the method of proof by contradiction: (8)
$p \rightarrow (q \wedge r)$, $r \rightarrow s$, $\neg(q \wedge s)$ Therefore $\neg p$.

(b) For the universe of all integers, let p(x), q(x), r(x), s(x), t(x) be the following open statements. (6)

p(x): x>0; q(x): x is even; r(x): x is a perfect square.

s(x): x is divisible by 4; t(x): x is divisible by 5.

Write the following statements in symbolic form.

(i) At least one integer is even.
(ii) There exists a positive integer that is even.
(iii) If x is even, then x is not divisible by 5.
(iv) If x is even and x is a perfect square, then x is divisible by 4.

Module 2

13(a) Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where (8)

(i) $x_i \geq 0$ for all i (ii) $x_i > 0$ for all i (iii) $x_1, x_2 \geq 5$, $x_3, x_4 \geq 7$.

(b) A tape contains 500,000 words of 4 or fewer lower case letters. Can it be that all of them are distinct? Use the pigeon hole principle. (6)

14(a) (i) Find the number of ways in which five persons can sit in a row. (8)

(ii) How many ways are there if two of the five persons insist on sitting next to one another

(iii) Solve part (i) assuming they sit around a circular table.

(iv) Solve part (ii) assuming they sit around a circular table.

(b) Find the coefficient of v^2w^4xz in the expansion of $(3v + 2w + x + y + z)^8$ (6)

Module 3

15(a) The relation R on the set of all integers Z is defined by aRb if $a-b$ is a non-negative even integer. Verify that R defines a partial order for Z. Is the partial order, a total order ? (8)

(b) If A= {1, 2, 3, 4, 5} and R is the equivalence relation on A that induces the partition A={1, 2}$\cup${3, 4}$\cup${5}, what is R? (6)

16(a) Draw the Hasse diagram for D_{24}, the set of all positive integer divisors of 24. Is $(D_{24}, |)$ a lattice under divisibility relation ? Substantiate. Are $B_1=\{1, 2, 3, 6, 12\}$ and $B_2=\{1,2,3,4,6,8,12\}$ sublattices of D_{24} ? (8)

(b) Define a complete lattice. Give an example each for a complete lattice and a lattice which is not complete. (6)

Module 4

17(a) Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ where n $\geq$ 2 and $a_0 = -1, a_1 = 8$. (8)

(b) In how many ways can two dozen identical robots be assigned to 4 distinct assembly lines with at least 3 robots assigned to each line? (6)

18(a) Solve the recurrence relation $a_{n+2} - 10a_{n+1} + 21a_n = 6(3)^n$. (8)

(b) Find the unique solution of the recurrence relation $4a_n - 5a_{n-1} = 0$; $n \geq 1$ (6)

Module 5

19(a) Prove that If H is a non-empty subset of a group G, then H is a subgroup of G if and only if it satisfies both (i) and (ii). (8)

(i) for all a, b $\in$ H, ab$\in$H (ii) for all a$\in$H, $a^{-1}\in$H.

(b) Define (i) a semigroup, a monoid. (ii) Give one example each for a semigroup which is not a monoid, and a monoid which is not a group. (6)

20(a) Let Z_n be the group of all integers mod n under modular addition. List (i) all generators of Z_9 (ii) all cyclic subgroups of Z_9, with their order. (8)

(b) Assuming Lagrange's theorem, prove that every group of prime order is cyclic. (6)

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