

H1

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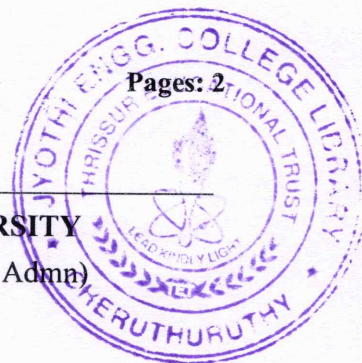
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Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S4 (Hons.) Degree Examination May 2025 (2023 Admn)



Course Code: CST292

Course Name: Number Theory

Max. Marks: 100

Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks)

Marks

- | | | |
|----|--|---|
| 1 | Show that $\mathbb{Z}_5 - \{0\}$ under multiplication modulo 5 forms a group. | 3 |
| 2 | Evaluate $\text{GCD}(2076, 1776)$ using Euclidean algorithm | 3 |
| 3 | Solve the congruence $12x \equiv 48 \pmod{18}$ | 3 |
| 4 | Show that 11 is prime using Wilson's theorem. | 3 |
| 5 | Is 10585 a Carmichael number? | 3 |
| 6 | Find the orders of the elements of U_5 | 3 |
| 7 | Determine if 3 is a quadratic residue of 29 using Euler's criterion. | 3 |
| 8 | Determine whether the Möbius function $\mu(mn) = \mu(m)\mu(n)$ where $m=15$ and $n=28$. | 3 |
| 9 | Prove that 221 is a sum of two squares. | 3 |
| 10 | Find the finite continued fraction of $\frac{57}{23}$ | 3 |

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

- | | | |
|----|--|---|
| 11 | a) Let $R = \{0, 2, 4, 6, 8\}$ under addition and multiplication modulo 10. Prove that R is a field. | 6 |
| | b) Find the general solution of the Diophantine equation $1485x + 1745y = 15$. | 8 |

OR

- | | | |
|----|---|---|
| 12 | a) Calculate the least absolute residue of $19 \times 14 \pmod{23}$. | 6 |
| | b) State Bezout's identity. Using the identity find the solution of $4076x + 1024y = 4$ | 8 |

Module -2

- 13 a) Using Fermat's Little Theorem find the remainder when 24^{1947} is divided by 17 7
 b) Define Fermat primes. Show that 257 and 65537 are mutually coprime. 7

OR

- 14 a) State Chinese Remainder Theorem. Solve the linear system 7

$$x \equiv 1 \pmod{3}$$

$$x \equiv 4 \pmod{5}$$

$$x \equiv 6 \pmod{7}$$

- b) Compute the remainder when 15^{1976} is divided by 23 7

Module -3

- 15 a) Find $\phi(14)$ and verify Euler's theorem for each 'a' coprime to 14. 6
 b) Define a pseudo prime to a base and find all non-trivial bases for which 15 is a pseudo prime. 8

OR

- 16 a) Using Euler's theorem find the ones digit in the decimal value of 17^{6666} 6
 b) Verify that 5 is a primitive root of U_7 and hence show that it generates elements of U_7 . 8

Module -4

- 17 a) Compute the Legendre $\left(\frac{3797}{7297}\right)$ 6
 b) Using Gauss's lemma, evaluate $\left(\frac{7}{19}\right)$. Also show that F_2 is prime using Pepin's test. 8

OR

- 18 a) Find the quadratic residues and non-residues of $p = 13$. 6
 b) State the generalised law of reciprocity and hence evaluate the Jacobi $(221|399)$ 8

Module -5

- 19 a) Express the integer 247 as sums of four squares. 7
 b) Define Gaussian integer. Prove that the set of Gaussian integers form a ring. 7

OR

- 20 a) Find all solutions of the Pell's equation $x^2 - 2y^2 = 1$ 7
 b) Define sum of two squares with illustration. Prove that the product of two sums of squares is the sum of two squares. 7