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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S4 (Hons.) Degree Examination May 2025 (2023 Admn)

Course Code: CST294

Course Name: Computational Fundamentals for Machine Learning

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	Prove that the vectors (1, 3, 1), (2, 3, -1) and (1, 2, -1) form a basis for R^3.	3
2	Consider the mapping ϕ: $R^3 \rightarrow R^4$ as $(x_1 - x_3, x_1 + x_2, x_3 - x_2, x_1 - 2x_2)$ for all $x = (x_1, x_2, x_3)$ in R^3. Then find ϕ (1, −2, 3).	3
3	Find the l_1 and l_2 norms of the vector$v = (-7,2,-1)$.	3
4	Find the inverse of A=$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$	3
5	Determine the first, second, third and fourth derivative of $V(x) = x^3 - x^2 + x - 1$	3
6	Write any three identities involving gradients.	3
7	Two fair dice are thrown simultaneously. The probability that the sum of numbers on the dice exceeds 8 is.	3
8	Define sample space, event and random variable.	3
9	What is gradient descent with momentum? How is the next update during each iteration determined?	3
10	Explain Stochastic Gradient Descent with momentum?	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Solve the system of equations by Gauss Elimination method. — 7

x+y+z=3

x+2y+3z=4

x+4y+9z=6

b) Find all solution of the following system of linear equations. — 7

$x_1+8x_3-4x_4 = 42$

$0x_1+x_2+2x_3+12x_4 = 8.$

12 a) Find Im(f) and Ker(f), when f: $R^3 \rightarrow R^3$ is given by f(a,b,c,d) = (a+b, b+c, a+c) — 7

b) Find a basis for row space, column space and null space of the matrix.

$$A=\begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$$ 7

Module -2

13 a) Find the Cholesky decomposition of the matrix. $\begin{bmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{bmatrix}$ 5

b) Describe Gram-Schmidt process.
Ortho-normalize the basis b1=(1,1,0), b2=(1,0,1) 9

14 a) Show that the set of vectors (1, 0,-1) $(1,\sqrt{2},1)$ $(1,-\sqrt{2},1)$ is mutually orthogonal. Construct Orthonormal of the vectors. 5

b) Explain the concept of SVD and find the SVD of the matrix $A=\begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$ 9

Module -3

15 a) What is automatic differentiation? Illustrate the same by differentiating the function $f(x) = \sqrt{x^2 + exp(x^2)} + cos\left(x^2 + exp(x^2)\right)$ where "exp" denotes the exponential function. Clearly draw the computation graph and explain the working. 8

b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$ prove that, 6

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right)$$

16 a) If $u = xlogxy$ where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$. 3

b) Expand $e^x log(1+y)$ in powers of x and y up to the terms of third degree. 5

c) With necessary equations and illustrations, explain how gradients are computed in a deep network. 6

Module -4

17 a) The probability that a cell in a wireless system is overloaded is $\frac{1}{3}$. When the cell is overloaded the probability that a cell is blocked is 0.3. If it is not overloaded the probability that a call is blocked is 0.1. (i) If you make a call what is the probability that it is blocked? (ii) If your call is actually blocked, what is the probability that it is due to the cell being overloaded? 7

b) Let X be a random variable taking values 1, 2, 3 and 4 with probabilities 1/6, 2/6, 2/6, 1/6 respectively. Calculate the mean and variance of random variables Y = 2X+3. 7

18 a) The possible values of a random variable X are 0, 1, 2 and 3. If P(X=2) = 2P(X=0) = 4P(X=3) and 4P(X =1) = 3P(X=2), find the PMF and CDF of X. Also calculate $P(X\neq0)$ 7

b) A firm has a service contract on its copier. The firm pays a flat rate of Rs.250 each month plus Rs.50 for each service visit. Let X denote the number of repairs during a month. From experience, the firm knows that the probability distribution of X is as follows: 7

x	0	1	2	3
$P(X = x)$	0.5	0.3	0.15	0.05

(i) What is the probability that the service cost next month would exceed Rs. 300?

(ii) What is the average service cost per month?

Module -5

19 a) By the method of least square find the best fitting straight line to the given data. 7

X	Y
5	15
10	19
15	23
20	26
25	30

b) Solve linear programming problem using simplex method. 7

Max $Z= 5x_1+3x_2$

subject to the constraints:

$x_1+ x_2 \leq 2$

$5x_1+ 2x_2 \leq 10$

$3x_1+ 3x_2 \leq 12$

$x_1, x_2 \geq 0$

20 a) A manufacture of furniture makes two products, chairs and tables. Processing of these products is done on two machine A and B. A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 30 hours on machine B. Profit gained by the manufacturer from a chair is Rs. 1 and from a table is Rs. 5 respectively. Formulate the problem a LPP in order to maximise the total project. 7

b) Find the maximum and minimum values of $f(x, y) = y^2- 4x^2$ subject to $x^2+2y^2=4$ using Lagrange Multipliers Solution. 7

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