A MAT101-12190X Pages: 4

Reg No.:_______________ Name:___________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

B.Tech Degree S1 (S,FE) S2 (S,FE) Examination May 2025 (2019 Scheme)



**Course Code: MAT101**

**Course Name: LINEAR ALGEBRA AND CALCULUS**

**(2019-Scheme)**

Max. Marks: 100 Duration: 3 Hours

## PART A

***Answer all questions, each carries 3 marks.***

| | | |
|---|---|---|
| 1 | Find the rank of the matrix $\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$ | (3) |
| 2 | Write down the matrix of the quadratic form $2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8xz$ | (3) |
| 3 | If $w = tan^{-1}(xyz)$, find the differential $dw\ at$ (1,1,1). | (3) |
| 4 | Find the slope of the function $f(x,y) = ycos(xy) + sin(xy)$ at $(\pi,1)$ along the $x$- direction. | (3) |
| 5 | Evaluate $\int_{-1}^{2}\int_{0}^{2}\int_{0}^{1}(x^2 + y^2 + z^2)dxdydz$ | (3) |
| 6 | $\iint_R (xsin\, y - ysinx)\, dA$ over the region $R = \{(x,y),\ 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}\}$ | (3) |
| 7 | Test the convergence of the series $\sum_{k=1}^{\infty} \frac{2k+1}{(k+1)^2}$ | (3) |
| 8 | Check the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{2^k}$ | (3) |
| 9 | Find the Taylors series for $f(x) = \sin \pi x$ about $x = \frac{1}{2}$ upto fourth degree | (3) |

term.

10 Find the Fourier constant $a_n$ for the function $f(x) = x^2$ in the interval $(-\pi, \pi)$. (3)

## PART B

***Answer one full question from each module, each question carries 14 marks***

### Module-I

11 a) Solve the following linear system of equations using Gauss elimination method. (7)

$$4y + 4z = 24$$

$$3x - 11y - 2z = -6$$

$$6x - 17y + z = 18$$

b) Find the eigenvalues and eigenvectors of (7)

$$\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

12 a) Find the matrix of transformation that diagonalize the matrix (7)

A= $\begin{bmatrix} -1 & 2 & -2 \\ 2 & 4 & 1 \\ 2 & 1 & 4 \end{bmatrix}$. Also find the diagonal matrix.

b) Find the canonical form corresponding to the quadratic form (7)

$8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$. Also find the definiteness.

### Module-II

13 a) Find the local linear approximation L to the function $f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$ at the point P (4,3). Compare the error in approximating $f$ by L at the point Q(3.92, 3.01). (7)

b) Find the absolute maximum and minimum values of (7)

$f(x,y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0,0),(3,0),(0,5).

14 a) If $w = r^2 - r\tan\theta$, $r = \sqrt{s}$, $\theta = \pi s$ at $s = \frac{1}{4}$, evaluate $\frac{dw}{ds}$ at $s = \frac{1}{4}$. (7)

b) The focal length of a mirror is given by $\frac{2}{f} = \frac{1}{v} - \frac{1}{u}$. Use differentials to find the percentage error in $f$ if $u$ and $v$ are both in error by 2% each. (7)

## Module-III

15 a) Change the order of integration and hence evaluate $\int_0^4 \int_y^4 \frac{x}{x^2+y^2}\,dx\,dy$ (7)

b) Find the total mass and center of gravity of the lamina with density $\delta(x,y) = xy$ in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the coordinate axes. (7)

16 a) Use double integral to find the volume of the solid bounded by the cylinder $x^2 + y^2 = 16$ and the planes $z = 0$, $z = 4 - x$ (7)

b) Evaluate $\iint_R y\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = 25$ and the line $x + y = 5$. (7)

## Module-IV

17 a) Find the general term of the series $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots\ldots.$ and use the ratio test to show that the series converges. (7)

b) Test the absolute or conditional convergence of $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^3+1}$ (7)

18 a) Discuss the convergence of $\sum_{k=1}^{\infty} \frac{x^k}{k^3+1}$ (7)

b) Verify the convergence of the series $\sum_{k=0}^{\infty} \left( \frac{5^k + 1}{3^k} \right)$ (7)

**Module-V**

19 a) Expand the function as a Fourier series (7)

$$f(x) = x - x^2, \qquad -1 < x < 1$$

b) Find the half range cosine series for ` $f(x) = x(\pi - x),\ 0 < x < \pi$ . (7)

20 a) Find the Fourier series of $f$ defined by $f(x) = \begin{cases} 1+x, & -\pi < x < 0 \\ 1-x, & 0 < x < \pi \end{cases}$ (7)

b) Obtain Fourier sine series for the function $f(x) = \cos x,\ 0 < x < \pi$ (7)

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