Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S4 (Minor) Degree Examination May 2025 (2023 Admission)

Course Code: CST284

Course Name: Mathematics for Machine Learning

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks)

		Marks
1	Define linear independence of a set of vectors. Find if $\{(1,1,1,1)^T, (-1,1,-1,1)^T, (1,1,-1,-1)^T\}$ is linearly independent in vector space R^4.	3
2	Show that the set V of all n x n matrices with trace zero is a subspace of the vector space of all n x n matrices .	3
3	What is norm on a vector space? Give an example.	3
4	Define orthogonal projection of a vector. Find the projection matrix P onto the line through the origin spanned by $b = [1\ 2\ 2]^T$	3
5	Find the Taylor Polynomial of degree 4 for $f(x) = x^4$ at x=1.	3
6	Define the Jacobian of a vector valued function from R^m to R^n	3
7	What is the difference between discrete and continuous random variables? Give an example.	3
8	State the sum rule and product rule for two random variables.	3
9	How change in step size will affect the gradient descent algorithm?	3
10	What is convex optimisation? Explain strong duality in convex optimisation.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) For a vector subspace $U \subseteq R^5$, spanned by the vectors 7

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \\ -1 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \\ -2 \end{bmatrix}, x_3 = \begin{bmatrix} 3 \\ -4 \\ 3 \\ 5 \\ -3 \end{bmatrix}, x_4 = \begin{bmatrix} -1 \\ 8 \\ -5 \\ -6 \\ 1 \end{bmatrix},$$ find a basis and dimension.

b) The mapping ϕ: $R^4 \to R^3$ *defined* by $\phi(x,y,s,t) = (x-y+s+t, x+2s-t, x+y+3s-3t)$. *Find the* basis and dimension of the range and kernel of ϕ. 7

12 a) Find the solution space of $x_1+2x_2-x_3+2x_4=0$, $2x_1+4x_2+14x_4=0$, $-x_1-2x_2+4x_3+13x_4=0$. 7

b) Consider a linear mapping $\phi : R^3 \to R^4$ whose transformation matrix is $A_\phi = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 3 \\ 3 & 7 & 1 \\ -1 & 2 & 4 \end{bmatrix}$ *with* respect to standard bases $B=\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$, $C=\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$. The formula for transformation matrix with respect to new bases $\tilde{B}=\left\{\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix}\right\}$, $\tilde{C}=\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\\1\end{bmatrix}\right\}$ *is* $T^{-1}A_\phi S$.

Find the matrices T and S. 7

Module -2

13 a) Explain the steps only for Gram-Schmidt algorithm to orthogonalize the basis $\{(1,-1,1),(1,0,1),(1,1,2)\}$. 7

b) *Find* the Cholesky factorisation : $A = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 8 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ 7

14 a) Diagonalise : $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ 7

b) Write a short note on SVD. What is rank-1 approximation? 7

Module -3

15 a) If $f(x_1,x_2) = e^{x_1 x_2^2}$, $g(t) = \begin{bmatrix} t\cos t \\ t\sin t \end{bmatrix}$, *define* $h(t)=(f \circ g)(t)$, *write* down the formula for $\frac{dh}{dt}$ using chain rule. 7

b) Explain back propagation algorithm. 7

16 a) Explain Automatic Differentiation. 7

b) *If* $f(x)=\sqrt{x^2+e^{x^2}}+\cos\left(x^2+e^{x^2}\right)$, show the computation graph. 7

Module -4

17	a)	Discuss Gaussian distribution and its properties.	7
	b)	Define covariance matrix of an n-dimensional random variable and state two of its properties.	7
18	a)	Define conjugate prior and state Baye's theorem. What is likelihood?	7
	b)	If $f(x)=3x^2, 0 \le x \le 1$ is the pdf of random variable, find the p d f of $Y=X^2$.	7

Module -5

19	a)	What is gradient descent algorithm? Explain gradient descent with momentum.	7
	b)	Write down only the steps to find the maximum and minimum values of the function $f(x,y,z) = x^2+y^2+z^2$ subject to the constraints $z^2 = x^2+y^2, x+y-z+1=0$ using Legrange multiplier method.	7
20	a)	Write down the dual: Maximise $z=x_1+2x_2$, subject to $-x_1+2x_2 \le 8$, $x_1+2x_2 \le 12$, $x_1-x_2 \le 3$, $x_1, x_2 \ge 0$.	7
	b)	Differentiate batch gradient descent and stochastic gradient descent. Write the advantages and disadvantages of each of them .	7

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