Reg No.:_______________ Name:______________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S2 (R) Examination May 2025 (2024 Scheme)

Course Code: PCCST205

Course Name: - DISCRETE MATHEMATICS

Max. Marks: 60 Duration: 2 hours 30 minutes

PART A

(Answer all questions. Each question carries 3 marks) CO Marks

		CO	Marks
1	If A={1,2,3,4}, B={3,4,5,6}, and U={1,2,3,4,5,6,7,8}, find the following using set operations: a) $A \cup B$ b) $A \cap B$ c) $(A \cup B)'$	CO1	(3)
2	Let $f(x)=x^2$ and $g(x)=x+1$. Find $(f\circ g)(x)$and $(g\circ f)(x)$. Are they equal?	CO1	(3)
3	Show that the following logical expressions are equivalent using truth tables: $\neg(p\wedge q)\equiv\neg p\vee\neg q$	CO2	(3)
4	What is proof by contradiction? Illustrate with a simple example.	CO2	(3)
5	Find the sequence for the generating function $f(x)=\frac{1}{1+2x}$.	CO3	(3)
6	A population of rabbits triples every month. If the initial population is 2, write a recurrence relation and find the population after 4 months.	CO3	(3)
7	Is $\mathbb{R}-\{0\}$, a group under multiplication? Justify.	CO4	(3)

8		Let S_3 be the set of all permutations of 3 elements. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ be two permutations in S_3. Find the composition $\sigma \circ \tau$, and determine if permutation composition is commutative in this case.	CO4	(3)

PART B

(Answer any one full question from each module, each question carries 9 marks)

Module -1

9	a)	Find the number of integers between 1 and 10000 inclusive which are divisible by none of 5, 6 or 8.	CO1	(5)
	b)	Show that the set Z of integers has cardinality $\aleph_0$.	CO1	(4)
10	a)	Define $R: Z \rightarrow Z$ by $R = \{(x, y) / 3\ divides\ x - y\}$. Prove that R is an equivalence relation on Z. Also, determine the equivalence classes of R.	CO1	(5)
	b)	Prove that $(S_{30}, \backslash)$ is a lattice. Also, draw the Hasse diagram.	CO1	(4)

Module -2

11	a)	Translate the following English sentence into logical notation: "Every student in the class has submitted the assignment." Then write its negation in symbolic form and express the negation in English.	CO2	(5)
	b)	Show that in any set of 10 distinct integers chosen from the numbers 1 to 18, there must be at least two numbers whose difference is exactly 9. State and use the Pigeonhole Principle in your explanation.	CO2	(4)
12	a)	Check the validity of the argument. "If horses or cows eat grass, then mosquito is the national bird. If mosquito is the national bird then peanut butter tastes good on hot-dogs. But peanut butter tastes terrible on hot-dogs. Therefore, cows didn't eat grass."	CO2	(5)
	b)	Explain contradiction method of proof of the statements. Give a proof by contradiction of the statement: "If 3n + 2 is odd, then n is odd".	CO2	(4)

Module -3

13	a)	Identify the generating function and solve the recurrence relation using generating functions: $a_n = 3a_{n-1} + 2, n \geq 1, a_0 = 1$.	CO3	(5)
	b)	Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every non – negative integer n.	CO3	(4)
14	a)	Solve the recurrence relation: $a_n = 3a_{n-1} + 2^n$, with $a_0 = 0$. What is the order of the recurrence relation?	CO3	(5)
	b)	What are the steps in strong mathematical induction? How is it different from ordinary induction?	CO3	(4)

Module -4

15	a)	Let $G = (Z, +)$. Show that the set $H = \{nk/k \in Z\}$ is a subgroup of G, where n is a fixed integer.	CO4	(5)
	b)	Define group homomorphism. Let $f: (R, +) \rightarrow (R^+, \times)$ be defined by $f(x) = e^x$. Show that f is a group homomorphism.	CO4	(4)
16	a)	Find all subgroups of the cyclic group Z_{12}, and identify their generators.	CO4	(5)
	b)	State and prove Lagrange's theorem.	CO4	(4)

***