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**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

B.Tech Degree S8 (R,S) Exam April 2025 (2019 Scheme)

**Course Code: CST454**

**Course Name: FUZZY SET THEORY AND APPLICATIONS**

**Max. Marks: 100** **Duration: 3 Hours**

**PART A**

***Answer all questions, each carries 3 marks.*** Marks

1 Given two fuzzy sets *A* and *B* with the following membership functions: (3)

$$\mu_A(x) = \begin{cases} 0.3 & x=1 \\ 0.7 & x=2 \\ 1.0 & x=3 \\ 0.6 & x=4 \end{cases}, \quad \mu_B(x) = \begin{cases} 0.5 & x=1 \\ 0.4 & x=2 \\ 0.8 & x=3 \\ 0.9 & x=4 \end{cases}$$

Calculate the union and intersection of A and B.

2 Two fuzzy sets P and Q are defined on X as follows: (3)

| μ(X) | X1 | X2 | X3 | X4 | X5 |
|---|---|---|---|---|---|
| P | 0.1 | 0.2 | 0.7 | 0.5 | 0.4 |
| Q | 0.9 | 0.6 | 0.3 | 0.2 | 0.8 |

According to λ-cut method, what is $(P \cap Q)_{0.4}$

3 Given two fuzzy relations *R* and *S*, where (3)

$$R = \begin{bmatrix} 0.2 & 0.5 & 0.8 \\ 0.6 & 0.7 & 0.3 \end{bmatrix}, \quad S = \begin{bmatrix} 0.9 & 0.4 \\ 0.5 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$$

Calculate max-min and max-product composition.

4 Three variables of interest in power transistors are the amount of current that can be switched, the voltage that can be switched, and the cost. The following membership functions for power transistors were developed from a hypothetical components catalog: (3)



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$$\text{Average current (in amperes)} = \underset{\sim}{I} = \left\{\frac{0.4}{0.8} + \frac{0.7}{0.9} + \frac{1}{1} + \frac{0.8}{1.1} + \frac{0.6}{1.2}\right\}.$$

$$\text{Average voltage (in volts)} = \underset{\sim}{V} = \left\{\frac{0.2}{30} + \frac{0.6}{45} + \frac{1}{60} + \frac{0.9}{75} + \frac{0.7}{90}\right\}.$$

Find the fuzzy Cartesian product between Average current and Average voltage using mamdani implication.

5 Define (i) Convex subnormal fuzzy set (ii) Concave normal fuzzy set. (3)

6 Given two datapoints, illustrate how max-min similarity between them can be computed. (3)

7 Compare Mamdani and Sugeno inference system. (3)

8 Explain the role of fuzzy implication in approximate reasoning. Provide examples of different fuzzy implication operators. (3)

9 Explain how fuzzy sets define decision boundaries in classification tasks. How does the fuzziness of these boundaries affect classifier performance? (3)

10 Describe how membership degrees are calculated in fuzzy clustering. Compare this process with traditional crisp clustering methods. (3)

**PART B**

***Answer any one full question from each module, each carries 14 marks.***

**Module I**

11 a) For steel design, the cross-sectional area to column-height ratio largely determines the susceptibility of the columns to buckling under axial loads. The normalized ratios are on the universe, X = {0, 1, 2, 3}. These ratios are characterized as "small" to "large" as follows (10)

$$\text{"Small"} = \left\{\frac{1}{0} + \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3}\right\}.$$

$$\text{"Large"} = \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3}\right\}.$$

**Find the following**

i. the algebraic sum and bounded difference of "Small" and "Large" fuzzy set Small

ii. the core, height, boundary, crossover point and scalar cardinality of fuzzy set small.

iii. Verify De Morgan's law for the fuzzy set *'Small'* and '*Large*'



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b) Given the fuzzy relation $R$ between sets $X$ and $Y$, where $X= \{a, b, c\}$ and $Y= \{x, y, z\}$, is defined by the following matrix: (4)

$$R = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.7 & 0.2 & 0.3 \\ 0.6 & 0.8 & 0.4 \end{bmatrix}$$

Calculate the Domain, Range and Inverse of $R$

**OR**

12 a) Let X be the universe of excess water-height to level-height ratios (percentage), X = {0.5, 0.75, 1.0, 1.75} and let Y be a universe of damage indices (million dollars), Y = {0, 0.5, 1.0, 7.0}. Suppose we have fuzzy sets for a given water-height ratio (WH~) and a given damage in millions (D ~), as follows: (8)

$$\mu_{\underset{\sim}{WH}}(x)=\left\{\frac{1.0}{0.5}+\frac{1.0}{0.75}+\frac{0.6}{1.0}+\frac{0.1}{1.75}\right\},$$ moderate water-height ratio (percentage).

$$\mu_{\underset{\sim}{D}}(y)=\left\{\frac{0.2}{0}+\frac{0.3}{0.5}+\frac{0.8}{1.0}+\frac{1.0}{7}\right\},$$ relatively large damage (million dollars).

i. Find the relation between water-height ratio and relatively large damage.

ii. Suppose we are given a new water-height ratio (WH~) as follows:

$$\mu_{\underset{\sim}{WH'}}(x)=\left\{\frac{0.0}{0.5}+\frac{1.0}{0.75}+\frac{0.7}{1.0}+\frac{0.4}{1.75}\right\}.$$

Using max–min composition, find the damage associated with this new water-height ratio.

b) Illustrate Excluded middle axioms of crisp and fuzzy set. (6)

**Module II**

13 a) A physician analyzes different samples of an infected organ (S) and classifies it as not infected, moderately infected, and seriously infected. Five samples were analyzed as given in the table below: (9)



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| Samples | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|---|---|---|---|---|---|
| Not infected | 0.6 | 0.3 | 0.1 | 0.9 | 0.8 |
| Moderately infected | 0.4 | 0.5 | 0.3 | 0.1 | 0.1 |
| Seriously infected | 0.0 | 0.2 | 0.6 | 0.0 | 0.1 |

Use the max–min method to find the similarity relation.

b) Explain the procedure to find the max-min transitive closure of the given relation with an example. (5)

**OR**

14 a) In the manufacture of concrete, there are two key variables: the water content (measured in percentage of total weight), and the temperature at curing in the batch plant (measured in degrees Fahrenheit). Each variable is characterized in fuzzy linguistic terms as follows: (9)

$$\text{"low temperature"} = \left\{\frac{1}{40} + \frac{0.7}{50} + \frac{0.5}{60} + \frac{0.3}{70} + \frac{0}{80}\right\},$$

$$\text{"High temperature"} = \left\{\frac{0}{40} + \frac{0.2}{50} + \frac{0.4}{60} + \frac{0.7}{70} + \frac{1.0}{80}\right\}$$

$$\text{"High water content"} = \left\{\frac{0}{1} + \frac{0.2}{2} + \frac{0.4}{3} + \frac{0.9}{4} + \frac{1}{5}\right\},$$

$$\text{"Low water content"} = \left\{\frac{1}{1} + \frac{0.8}{2} + \frac{0.6}{3} + \frac{0.4}{4} + \frac{0.2}{5}\right\}$$

(I) Find the membership functions for the following phrases

(i) Water content not very low or not very very high

(ii) Temperature not very low and not very high

(iii) Water content slightly high

$(II)$ According to λ cut method, what is (water content not very high $)_{0.4}$.

b) (i) Define fuzzy tolerance and equivalence relation. Check whether the given relation is fuzzy tolerance relation. (5)

$$R = \begin{bmatrix} 0.9 & 0.2 & 0.5 & 0.4 \\ 0.3 & 1.0 & 0.7 & 0.6 \\ 0.8 & 0.4 & 0.6 & 0.2 \\ 0.1 & 0.7 & 0.9 & 0.3 \end{bmatrix}$$

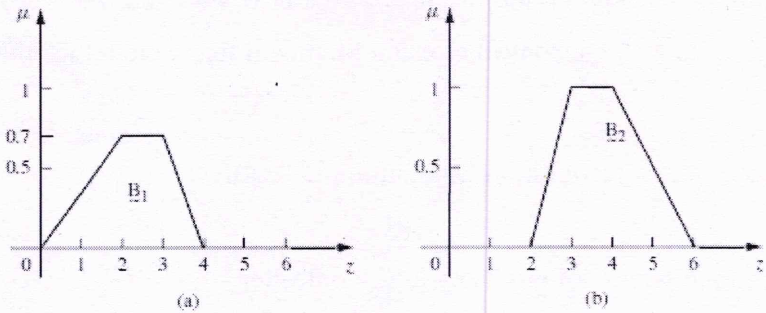
**Module III**

15 a) Illustrate weighted average method and centroid method of defuzzification. (8)

b) Using the inference approach, find the membership value for each of the triangular shapes (i) approximately isosceles triangle (ii) approximately equilateral triangle with angles 25°, 35°, 120°. (6)

**OR**

16 a) Check whether maxima and centre of sum method is applicable for the logical union of the membership functions shown below and, if so, calculate the defuzzified value, $z^*$. (8)




b) Explain Rank ordering and inference approach for membership value assignments with examples. (6)

**Module IV**

17 a) Consider the following two discrete fuzzy sets, which are defined on universe $X = \{-5, 5\}$: (9)

$$\underset{\sim}{A} = \text{“zero”} = \left\{ \frac{0}{-2} + \frac{0.5}{-1} + \frac{1.0}{0} + \frac{0.5}{1} + \frac{0}{2} \right\}.$$

$$\underset{\sim}{B} = \text{“positive medium”} = \left\{ \frac{0}{0} + \frac{0.6}{1} + \frac{1.0}{2} + \frac{0.6}{3} + \frac{0}{4} \right\}.$$

(a) Construct the relation for the rule IF A~, THEN B~ using zadeh's max min rule and product implication

(b) If we introduce a new antecedent

$$\underset{\sim}{A}' = \text{“positive small”} = \left\{ \frac{0}{-2} + \frac{0.1}{-1} + \frac{0.3}{0} + \frac{0.6}{1} + \frac{1}{2} \right\},$$

find the new consequent B~ using max–min composition

b) Describe Tsukamoto model for fuzzy inference system. List out its drawbacks. (5)

**OR**



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18 a) Illustrate how the Max-Min and Max-Product graphical inference techniques are applied using the Mamdani method. (8)

b) Explain the concept of fuzzy implication and aggregation of fuzzy rules in fuzzy logic. (6)

**Module V**

19 a) A metro train system uses fuzzy logic in ensuring smooth ride on the train. The system has fixed stops and the distance between the stops are known and it uses fuzzy logic in deciding the pressure applied on the brakes. The amount of pressure applied depends on the distance to the next stop and the speed of the train. Design a fuzzy logic controller using Mamdani fuzzy model for the above scenario. (10)

b) Explain fuzzy pattern recognition using multiple features. (4)

**OR**

20 a) Illustrate the architecture of the fuzzy logic controller. (7)

b) Explain Fuzzy C-means clustering with an example. (7)

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