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221TCS002012501 Pages: 2 Reg No .: Name: APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY M.Tech Degree S1 (R, S) / S1 (WP) (R, S) Examination December 2024 (2022 scheme) Course Code & Name: 221TCS002 FOUNDATIONS OF COMPUTER SCIENCE Max. Marks: 60 Duration: 2.5 Hours Normal Distribution tables are permitted during the examination PART A Answer all questions. Each question carries 5 marks Marks Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n (5)is a positive integer. 2 In how many ways 11 players can be selected out of 15 players, if (i) one (5) particular player is always selected, (ii) one particular player is never selected and (iii) there is no restriction on selection. Solve the recurrence relation: $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. (5) Let P(X = 2) = 9P(X = 4) + 90P(X = 6) where X is a Poisson Random (5) Variable. Find the mean and variance of the distribution. Consider the algebraic structure (A, *) with $A = \{a \in Q \mid a \neq -1\}$, and for any (5) $a, b \in A$, a * b = a + b + ab. Check whether (A, *) an abelian group or not.

PART B

Justify your answer.

Answer any 5 questions. Each question carries 7 marks

- State Pigeonhole principle. Using the principle, find the minimum number of (7)students required in a class to be sure that at least six will receive the same grade. (Note: there are only five possible grades: A, B, C, D and F).
- Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (7)
- 8 Determine number of positive integers, $0 \le n \le 720$, which are not divisible by (7)any of the numbers 2, 3, 5.
- Find the generating function for the sequence $\{a_n\}$ with (i) $a_n = n + 1$ and (7)(ii) $a_n = 3^n$

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- 10 State Baye's Theorem. There are two identical boxes containing respectively 4 (7) white and 3 red balls; 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball is white, what is the probability that it is from the first box?
- 11 If X is a normal random variable with parameters $\mu = 3$ and $\sigma^2 = 9$. Find: (7) (i) $P\{2 < X < 5\}$ (ii) $P\{X > 0\}$ and (iii) $P\{|X - 3| > 6\}$.
- 12 'The order of any element of a finite group is a divisor of the order of the group'prove the statement. (7)