C 15288 (Pages 4)

Name.................................

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FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2010

Computer Science Engineering

CS/IT/PTCS 2K 401—ENGINEERING MATHEMATICS—IV

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Find the complex exponential Fourier transform of $f(x) = \begin{cases} k & \text{in } |x| \le l \\ 0 & \text{in } |x| > l \end{cases}$.

(b) State and prove modulation theorem.

(c) Find 'k' for which the following is a probability mass function. Also find mean and variance.

x	:	0	1	2	3
$f(x)$	:	$k/2$	$k/3$	$(k+1)/3$	$(2k-1)/6$

(d) Derive the moment generating function of a Chi-square distribution.

(e) Find the probability distribution (X + Y) from the bivariate distribution of (X, Y) given below :

X \ Y	1	2
1	0.1	0.2
2	0.3	0.4

(f) If {X (s, t)} is a random process, what is the nature of X(s, t) when (i) S is fixed ; (ii) t is fixed.

(g) Explain the memoryless property of the exponential distribution.

(h) Define a Poisson process with a suitable example.

(8 × 5 = 40 marks)

II. (a) Express $f(x) = \begin{cases} 1/2, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence show that :

$$\int_0^\infty \frac{1 - \cos \pi\lambda}{\lambda} \sin x\lambda \, d\lambda = \begin{cases} 1/2, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}.$$

(7 marks)

Turn over

(b) Find the Fourier transform of :

$$f(x)=\begin{cases} a-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin^4 x}{x^4}\,dx=\frac{\pi}{3}$.

(8 marks)

(c) Find the Fourier transform of

$$f(x)=\begin{cases} a-|x| & \text{if} |x|<a \\ 0 & \text{if} |x|>a \end{cases}$$

(7 marks)

(d) Solve the differential equation :

$y''-4y'+5y=1,\ x>0$ give that $y(0)=0,\ y'(0)=0$.

(8 marks)

III. (a) A car hire firm has 2 cars which it hires out day by day. The number of demands for a car in each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of dates on which (i) there is no demand ; (ii) Some demand is refused.

(7 marks)

(b) If X follows a binomial distribution, show that $E\left(\frac{X-np}{\sqrt{npq}}\right)=0$ and $E\left(\frac{X-np}{\sqrt{npq}}\right)^2=1$.

(8 marks)

Or

(c) Fit a binomial distribution $B(n, p)$ for the following data :—

x	:	0	1	2	3	4	5	6	7
f	:	7	6	19	35	30	26	7	1

(7 marks)

(d) If the density function of a continuous r.v. X is given by :

$$f(x)=\begin{cases} ax, & 0\le x\le 1 \\ a, & 1\le x\le 2 \\ 3a-x, & 2\le x\le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) find the value of 'a' ;

(ii) find the cdf of X ;

(iii) If x_1, x_2, x_3 are 3 independent observations of X,

What is the probability that exactly one of these three is greater than 1.5 ?

(8 marks)

IV. (a) Given that the joint p.d.f. of (X, Y) is give by

$$f(x, y) = \begin{cases} e^{-y}, & x > 0,\ y > x \\ 0, & \text{else where} \end{cases}$$

find :

(i) P (X > 1/Y < 5) and

(ii) the marginal distribution of X and Y.

(7 marks)

(b) A distribution with unknown mean 'μ' has variance equal to 1.5. Use central limit theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean.

(8 marks)

Or

(c) If X and Y are independent random variables prove that E(Y/X) = E (Y) and E (X/Y) = E(X).

(7 marks)

(d) Given $f_{xy}(x, y) = c_x(x - y),\ 0 < x < 2,\ -x < y < x$ and 0 elsewhere.

(a) evaluate C ;

(b) find $f_x(x)$.

(c) $f_{y/x}(y/x)$.

(d) $f_y(y)$.

(8 marks)

V. (a) The TPM of a Markov chain $\{X_n\}$, $n = 1, 2, \ldots$ with 3 states 1, 2, and 3 is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

and the initial state distribution of the chain is $P[X_0 = i] = 1/3$, $i = 0, 1, 2$. Find (i) $P[X_2 = 2]$ (ii) $P[x_3 = 1, x_2 = 2, x_1 = 1, x_0 = 2]$.

(7 marks)

Turn over

(b) Prove that sum of 2 independent Poisson processes is a Poisson process. (8 marks)

Or

(c) Consider a Markov chain with states 0, 1, 2, 3, 4. Suppose $P_{0,4} = 1$ and suppose that when the chain is in the state, i, $i > 0$ the next step is equally likely to be any of the states 0, 1, 2, ... $i - 1$. Find the limiting probabilities of the Markov chain.

(7 marks)

(d) A gambler has Rs. 2. He bets Re.1 at a time and wins Re. 1 with probability 1/2. He stops playing if he loses Rs. 2 or wins Rs.4 :

(i) What is the TPM of the related Markov chain ?

(ii) What is the probability that he has lost his money at the end of 5 plays ?

(iii) What is the probability that the game lasts more than 7 plays ?

(8 marks)

[4 × 15 = 60 marks]