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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S, FE) / S4 (PT) (S,FE) / S2 (PT) (S,FE) / S4 (WP) (S) Examination December 2024 (2019 Scheme

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	The probability that there will be 0,1,2,3 defective light bulbs in a randomly selected batch of bulbs produced in a factory are 0.6, 0.3, 0.08, 0.02 respectively. Find the mean and variance of the number of defective bulbs in a randomly selected batch.	(3)
2	If X is a Poisson random variable with $E[X] = 6$, find $P(X = 4)$.	(3)
3	Consider a continuous random variable X with the probability density function $f(x) = 2x,\ 0 \le x \le 1$. Find $P(0.25 \le X \le 0.75)$ and $E[X]$.	(3)
4	For an exponential random variable X, given that $P(X > 2) = e^{-10}$. Find $P(X > 7/X > 5)$.	(3)
5	Find the variance at time $t = 3$ of a WSS random process $X(t)$ with autocorrelation function $R_X(\tau) = e^{-\|\tau\|}$.	(3)
6	Show that the power spectral density of a WSS process is a real valued even function.	(3)
7	Find the first iterate x_1 for the root of the function $f(x) = x^3 - 5x - 9$ using Newton-Raphson method with initial guess $x_0 = 2$.	(3)
8	Find the divided differences for the following data points (1,4), (2,7), (3,12), (4,19).	(3)
9	Write the normal equations for fitting a parabola of the form $y = a + bx + cx^2$ to a given data by the method of least squares.	(3)
10	Write any sufficient conditions for the Gauss-Seidel method for iterating the solution of a linear system to converge.	(3)

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) A factory produces electronic components, and historically, 5% of these components are defective. If a batch contains 200 components: (7)

(i) What is the probability that exactly 15 components are defective according to the binomial distribution?

(ii) Using the Poisson approximation to the binomial distribution, estimate the probability that the batch contains at most 5 defective components.

b) Find the mean and variance of a Poisson random variable with parameter λ. (7)

12 a) Let X be a discrete random variable with the following probability mass function (7)

$P(X = 1) = 0.3, P(X = 2) = 0.2,\quad P(X = 3) = 0.1,\quad P(X = 4) = 0.4$

(i) Verify that this function satisfies the properties of a PMF.

(ii) Calculate the probability that X is an even number.

(iii) Determine the cumulative distribution function (CDF) of X.

b) Let X and Y be discrete random variables with the following joint probability mass function (PMF): (7)

	$Y = -1$	$Y = 0$	$Y = 1$
$X = 1$	0.1	0.2	0.1
$X = 2$	0.2	0.15	0.25

(i) Calculate $P(X = 2, Y \leq 0)$

(ii) Find the marginal PMFs of X and Y.

(iii) Determine the expected values of X and Y.

Module -2

13 a) The heights of a population of adult males in a certain country follow a normal distribution with a mean of 175 cm and a standard deviation of 7 cm. (7)

(i) What percentage of adult males in this country have a height greater than 185 cm?

(ii) If 200 adult males are randomly selected from this population, what is the expected number of individuals with a height between 170 cm and 180 cm?

b) Passengers come to a bus station at random times. The waiting time of a random passenger, until his bus comes, follows a uniform distribution between 5 and 15 minutes. (7)

(i) Calculate the probability that a person waiting at this bus stop will wait for more than 10 minutes.

(ii) If 120 people arrive at this bus stop independently, what is the expected number of people who will wait between 7 and 12 minutes?

14 a) Random variables X and Y have joint probability density function (7)

$$f(x, y) = \begin{cases} k(x + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & otherwise \end{cases}$$

(i) Find the value of k so that $f(x, y)$ is a valid probability function.

(ii) Find $P\left(0 < X < \frac{1}{4}, 0 < Y < \frac{1}{4}\right)$.

(iii) Find the marginal density functions of X and Y.

b) A study on the average monthly internet usage in a small town shows a wide range of internet usage values with a mean of 250 GB and a standard deviation of 60 GB. What is the probability that for a random sample of 100 residents from this population (i) the mean monthly internet usage is 260 GB or more? (ii) the total monthly usage is less than 20000 GB . (7)

Module -3

15 a) A random process X(t) is defined by $X(t) = Acos(\omega t + \phi)$ where ω is a constant, A is a random variable which is uniformly distributed in [0,1] and ϕ is a random variable, independent of A and uniformly distributed in $[0, 2\pi]$. Determine whether the random process X(t) is Weakly Stationary (WSS) or not. (7)

b) The autocorrelation function of a continuous-time Weakly Stationary (WSS) process X(t) is $R_X(\tau) = \sigma^2 e^{-|\tau|}$, where σ^2 is the variance of the process. Find the power spectral density of the process and the average power of the process. (7)

16 a) If people arrive at a book stall in accordance with a Poisson process with mean rate of 3 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 minute (ii) between 1 minute and 2 minutes (iii) 4 minutes or less. (7)

b) Prove that autocorrelation of a random process is an even function. (7)

Module -4

17 a) Solve the equation $xe^x - 2 = 0$ using Regula Falsi Method. (7)

b) A third degree polynomial passes through the points (0,-1), (1,1), (2,1) and (3,-2). Using Newton's forward interpolation formula find the polynomial. Hence find the value at 1.5

18 a) Using Lagrange's interpolation method compute the value of y(4) for the following data (7)

x	1	3	5	7
y	24	120	336	720

b) Calculate $\int_{0.5}^{0.7} e^{-x}\sqrt{x}\, dx$ using Simpson's method by taking 5 ordinates. (7)

Module -5

19 a) Solve the following system of equations using Jacobi's method (7)

$$27x + 6y - z = 85, \quad x + y + 54z = 110, \quad 6x + 15y + 2z = 72$$

b) Using Euler's method, find $y(0.5)$ of $y(x)$ which satisfies the initial value problem, $\frac{dy}{dx} = \frac{1}{2}(x^2 + 1)y^2$, given $y(0.2) = 1.1114$. Take $h = 0.1$. (7)

20 a) Using Runge-Kutta method of order 2, evaluate y(0.1) if $\frac{dy}{dx} = x^2 + y^2$, given that $y(0) = 1$. (7)

b) Using Adams Moulton method find $y(4.4)$ for the differential equation $\frac{dy}{dx} = \frac{2-y^2}{5x}$. Given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$ (7)

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