

A

0200MAT256042501

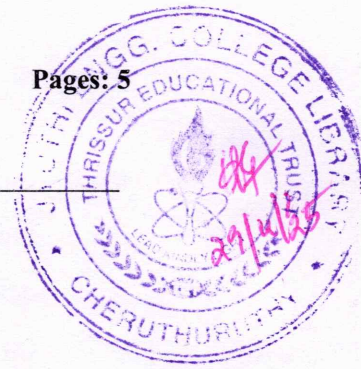
Pages: 5

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (R,S) Exam April 2025 (2019 Scheme)



Course Code:: MAT256

Course Name: PROBABILITY AND STATISTICAL MODELLING

Max. Marks: 100

Duration: 3 Hours

(Statistical tables are allowed)

## PART A

(Answer all questions; each question carries 3 marks)

Marks

- 1 Determine the binomial distribution for which the mean is 4 and the variance is 3. 3
- 2 If X is a Poisson variable such that  $P(X=1) = \frac{3}{10}$  and  $P(X=2) = \frac{1}{5}$ , find the mean. 3
- 3 Find the mean and variance of Exponential distribution. 3
- 4 If X has uniform distribution in  $(-3,3)$ , find  $P(|X-2| < 2)$ . 3
- 5 Describe Chi square distribution. 3
- 6 A sample of 16 measurements of the diameter of a sphere gives a mean of 4.58 inches and a standard deviation of 0.08 inches. Find a 95% confidence limit for the actual diameter. 3
- 7 Write a short note on one way ANOVA. 3
- 8 Define Null hypothesis and Alternating Hypothesis with an example. 3
- 9 Write a short note on Simple Linear Regression Model. 3
- 10 Calculate the regression coefficients and obtain the lines of regression for the following data 3

X	1	2	3	4	5
Y	2	5	11	8	14

## PART B

(Answer one full question from each module, each question carries 14 marks)

## Module -1



- 11 a) It is known that 5% of the books bound at a certain bindery have defective bindings. Find the probability that at most 2 of 100 books bound by this bindery will have defective binding using 7
- (a) The formula for binomial distribution.(b) Poisson approximation to binomial.
- b) The joint probability distribution of  $X$  and  $Y$  is given by  $f(x, y) = \frac{1}{27}(2x + y)$ ;  $x = 0, 1, 2$  and  $y = 0, 1, 2$  (i) Find the marginal distributions of  $X$  and  $Y$ .(ii) Are  $X$  and  $Y$  independent random variables? 7
- 12 a) Find  $a, b$  if  $Y = aX + b$  has mean 4 and variance 16, where  $X$  is a random variable with mean 8 and variance 4. 7
- b) Derive the formula for mean and variance of the Poisson distribution 7

### Module -2

- 13 a) The time (in hours) required to repair a machine is exponentially distributed with mean 2. 7
- (i) What is the probability that the repairing time exceeds 2 hours?
- (ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours?
- b) Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8, as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (Forest Products J., May 1997 : 36-41). 7
- (i) For what value of  $c$ , the interval  $(8.8 - c, 8.8 + c)$  includes 98% of all diameter values?
- (ii) If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 inches?
- 14 a) The amount of time that a surveillance camera will run without having to be reset is a random variable having exponential distribution with mean 50 days. Find the probabilities that such a camera will 7
- i. have to be reset in less than 20 days.ii. not have to be reset in at least 60 days.
- b) In an examination, 30% of the students got marks below 40 and 10% got marks above 75. Assuming the marks are normally distributed, find the mean and standard deviation of the distribution. 7

### Module -3



- 15 a) The specimen of copper wires drawn from a large lot have the following breaking strength (in kg. weight): 578, 572, 570, 568, 572, 578, 570, 572, 596, 544. Test whether the mean breaking strength of the lot may be taken to be 578 kg. weight. Test at 5 percent level of significance. (Use Student's t-statistic) 7
- b) What should be the size of the sample if a simple random sample from a population of 4000 items is to be drawn to estimate the per cent defective within 2 per cent of the true value with 95.5 per cent probability? What would be the size of the sample if the population is assumed to be infinite in the given case? 7
- 16 a) Determine the size of the sample for estimating the true weight of the cereal containers for the universe with  $N = 5000$  on the basis of the following information: 7
- (i) The variance of weight = 4 ounces on the basis of past records.
- (ii) Estimate should be within 0.8 ounces of the true average weight with 99% probability.
- Will there be a change in the size of the sample if we assume infinite population in the given case? If so, explain by how much?
- b) In a random selection of 64 of the 2400 intersections in a small city, the mean number of scooter accidents per year was 3.2 and the sample standard deviation was 0.8. 7
- (i) Make an estimate of the standard deviation of the population from the sample standard deviation.
- (ii) Work out the standard error of mean for this finite population.
- (iii) If the desired confidence level is 0.90, what will be the upper and lower limits of the confidence interval for the mean number of accidents per intersection per year?

#### Module -4

- 17 a) Six samples of each of 4 types of cereal grain grown in a certain region were analyzed to determine thiamine content, result in the following data. 7

Wheat	5.2	4.5	6	6.1	6.7	5.8
Barley	6.5	8	6.1	7.5	5.9	5.6
Maize	5.8	4.7	6.4	4.9	6	5.2
Oats	8.3	6.1	7.8	7	5.5	7.2



Does this data suggest that at least two of the grains differ with respect to true average thiamine content? Use level of significance  $\alpha = 0.05$

- b) A random sample of 150 recent donations at a certain blood bank reveals that 82 were type A blood. Does this suggest that the actual percentage of type A donations differs from 40%, the percentage of the population having type A blood? Carry out a test of the appropriate hypotheses using a significance level of 0.01.
- 18 a) Four different locations in the northeast were used for collecting ozone measurements in parts per million. Amounts of ozone were collected in 5 samples at each location. Is there sufficient information here to suggest that there are differences in the mean ozone levels across locations at 5% level of significance?

Location			
I	II	III	IV
0.09	0.15	0.10	0.10
0.10	0.12	0.13	0.07
0.08	0.17	0.08	0.05
0.08	0.18	0.08	0.08
0.11	0.14	0.09	0.09

- b) The mean lifetime of certain products is 1800 hrs with SD of 100 hrs. By applying a new technique, it is claimed that the mean life has increased. To test the claim, a sample of 50 products were taken and it is found that the mean life time is 1850 hrs. Can we support the claim at 1% level of significance?

#### Module -5

- 19 a) Suppose the variables  $x$ = commuting distance and  $y$ = commuting time are related according to the simple linear regression model with  $\sigma = 10$ . If  $n=5$  observations are made at the  $x$  values  $x_1= 5$ ,  $x_2= 10$ ,  $x_3= 15$ ,  $x_4= 20$ , and  $x_5=50$ . Calculate the standard deviations of the five corresponding residuals.
- b) Thermal endurance tests were performed to study the relationship between temperature and lifetime of polyester enamelled wire, resulting in the following data

Temp	200	200	200	200	200	200
Lifetime	5933	5404	4947	4963	3358	3878
Temp	220	220	220	220	220	220
Lifetime	1561	1494	747	768	609	777



## 0200MAT256042501

Temp	240	240	240	240	240	240
Lifetime	258	299	209	144	180	184

- a) What model is implied by a linear relationship between expected  $\log(\text{lifetime})$  and  $\frac{1}{\text{Temperature}}$ .
- b) Estimate the parameters of the model suggested in a).
- c) What lifetime would you predict for a temperature of 220.
- 20 a) The flow rate  $y$  (m'/min) in a device used for air-quality measurement depends on the pressure drop  $x$  (inches of water) across the device's filter. Suppose that for  $X$  values between 5 and 20, the two variables are related according to the simple linear regression model with true regression line  $y = -0.12 + 0.095x$  7
- (i) What is the expected change in flow rate associated with a 1-inch increase in pressure drop. 5-inch decrease in pressure drop?
- (ii) What is the expected flow rate for a pressure drop of 10-inch?
- iii) Suppose  $\sigma = .025$  and consider a pressure drop of 10-inch. What is the probability that the observed value of the flow rate will exceed 0.835?
- b) The Turbine Oil Oxidation Test (TOST) and the Rotating Bomb Oxidation Test (RBOT) are two different procedures for evaluating the oxidation stability of steam turbine oils. 7
- The article reported the accompanying data (read from a graph) on  $x$  shear force (kg) and  $y$  percent fibre dry weight.
- TOST 4200 3600 3750 3675 4050 2770
- RBOT 370 340 375 310 350 200
- TOST 4870 4500 3450 2700 3750 3300
- RBOT 400 375 285 225 345 285
- a. Calculate the value of the sample correlation coefficient.
- b. Carry out a test of hypotheses to decide whether RBOT time and TOST time are linearly related.

\*\*\*