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Reg No.:_______________ Name:_______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S, FE) S2 (S, FE) Examination December 2024 (2019 Scheme)

Course Code: MAT 101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

		Marks
1	Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$	(3)
2	What kind of conic section is given by the quadratic form $7x^2 + 6xy + 7y^2 = 200$.	(3)
3	Find the slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0)	(3)
4	Prove that $f_{xy} = f_{yx}$ where $f(x, y) = \ln(x^2 + y^2)$	(3)
5	Find the area bounded by the x-axis, $y = 2x$ and $x + y = 1$ using double integrals.	(3)
6	Evaluate $\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^3 dzdydx$	(3)
7	Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{1}{3}\right)^n$	(3)
8	Determine the rational number representing the decimal number 0.764764764….	(3)
9	Find the Maclaurin series expansion of $f(x) = \ln(1 - x)$ upto 3 terms	(3)
10	If $f(x)$ is a periodic function with period $2l$ defined in $[-l, l]$, write the expressions for the Fourier coefficients a_0, a_n, and b_n.	(3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11	a	Using the Gauss elimination method, solve the system of equations $x + 2y + z = 3, 2x + 3y + 2z = 5, 3x - 5y + 5z = 2, 3x + 9y - z = 4$	(7)
	b	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$	(7)

12	a	Test for consistency and solve the following system of equations: $2x - y + 3z = 8, -x + 2y + z = 4, 3x + y - 4z = 0$	(7)
	b	Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$	(7)

MODULE 2

13	a	If $u = f(x/y, y/z, z/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.	(7)
	b	Find relative extrema and saddle points, if any, of the function $f(x, y) = x^3 + y^3 - 15xy$	(7)
14	a	If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$	(7)
	b	Find the local linear approximation L to the function $f(x, y, z) = \log(x + yz)$ at the point $P(2,1,-1)$.Compute the error in approximation f by L at the point Q(2.02,0.97,-1.01).	(7)

MODULE 3

15	a	Evaluate $\iint^R y dA$ where R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.	(7)
	b	Evaluate the integral $\int_0^4 \int_y^4 \frac{x}{x^2+y^2} dxdy$ by first reversing the order of integration.	(7)
16	a	Find the center of gravity of the triangular lamina with vertices $(0,0), (0,1)$ *and* $(1,0)$ and density function $\delta(x, y) = xy$.	(7)
	b	Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the planes $y + z = 3$ and $z = 0$	(7)

MODULE 4

17	a	Check the convergence of the series $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \frac{3.4.5.6}{4.6.8.10} + \cdots \ldots \ldots$	(7)
	b	Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(k+3)}{k(k+1)}$ (ii) $\sum_{k=1}^{\infty} \frac{2}{3^k+5}$	(7)

18	a	Test the convergence of the series (i) $\sum_{k=1}^{\infty}\left(\frac{3k-4}{4k-5}\right)^k$ (ii) $\sum_{k=1}^{\infty}\frac{1}{2k^2+k}$	(7)
	b	Test the absolute or conditional convergence of $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}k^2}{k^3+1}$	(7)

MODULE 5

19	a	Find the Fourier series for $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi, & 0 < x < \pi \end{cases}$ and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \ldots\ldots.. = \frac{\pi}{4}$	(7)
	b	Find the half range Fourier sine series representation of $f(x) = e^x \; in \; (0,1)$	(7)
20	a	Find the Fourier series for $f(x) = \lvert sinx \rvert, -\pi < x < \pi$	(7)
	b	Obtain the half range Fourier cosine series of $f(x) = x^2$ in $0 < x < 2$	(7)

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