0200MAT204122305

Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (R,S) (FT/WP/PT) / (S2 PT) Exam April 2025 (2019 Scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	Given that $f(x) = \frac{k}{2^x}$ is a probability mass function of a random variable that can take values $x = 0,1,2,3$ and 4, Find (a) k (b) $P(X \leq 2)$.	3
2	The joint PMF of two random variables X and Y is given by $p(x, y) = c(x + y),\ x = 0,1,2;\ y = 0,1,2,3$. Find c	3
3	If X has uniform distribution in $(-a, a)$, find a such that $P(\lvert X\rvert \leq 1) = P(\lvert X\rvert > 1)$.	3
4	The lifetime (in years) of an electronic component is an exponential random variable with mean 1 year. Find the lifetime L which a typical component is 60% certain to exceed.	3
5	Find the power spectral density of the WSS power process with autocorrelation function $2e^{-\lvert\tau\rvert} + 4e^{-\lvert\tau\rvert}$.	3
6	Find the mean function of the sine wave random process with random amplitude defined by $X(t) = A\sin(\omega_0 t)$ where ω_0 is a constant and A is a random variable uniformly distributed in (0, 1).	3
7	Evaluate $\int_0^{\frac{\pi}{2}} \cos x\ dx$ using trapezoidal rule in 6 sub intervals.	3
8	Find an approximate root of the equation $x^3 - 2x - 5 = 0$ using Regula Falsi method.	3
9	Using Euler's method, find y at $x = 0.25$, given $y' = 2xy,\ y(0) = 1,\ h =$	3

159

0.25.

10 Write the normal equations for fitting a curve of the form $y = a + bx$ to a given set of data points. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) A random variable X has the following probability distribution 7

X	-2	-1	0	1	2	3
Y	$1/10$	$15k^2$	$1/5$	$2k$	$3/10$	$3k$

Find (a) k (b) $P(X < 2)$ (c) $P(X - 2 << 2)$ (d) Mean and variance of X.

b) Derive the mean and variance of Binomial distribution. 7

12 a) In a binomial distribution consisting of 6 independent trials, probabilities of 1 and 2 successes are 0.28336 and .0506 respectively. Find the mean and variance of the distribution. 7

b) Accidents occur at an intersection at a Poisson rate of 2 per day. What is the probability that in January there would be at least 3 days(not necessarily consecutive) without any accidents? 7

Module -2

13 a) The probability density function of a random variable X is given by 7

$$f(x) = \begin{cases} kx^3, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Find (a) $P(\frac{1}{4} < X < \frac{3}{4})$ (b) $P\left(X > \frac{1}{2}\right)$.

b) The marks obtained by a batch of students in a certain subject are normally distributed. 10% of students got less than 45 marks while 5% of students got more than 75. Find the percentage of students with score between 45 and 60. 7

14 a) The amount of time that a surveillance camera will run without having to be reset 7

is a random variable having exponential distribution with mean 50 days. Find the probability that such a camera will (a) have to be reset in less than 20 days (b) not have to be reset in at least 60 days.

b) The joint density function of 2 continuous random variables X and Y is 7

$$f(x,y) = \begin{cases} cxy, & 0 < x < 4,\ 1 < y < 5 \\ 0, & otherwise \end{cases}$$

a) Find the value of c.

b) Find $P(X \geq 3, Y \leq 4)$.

c) Find the marginal densities of X and Y.

Module -3

15 a) A random process is defined by $X(t) = 2\cos(5t + \theta)$ where θ is uniformly distributed in $[0,2\pi]$. Find the mean, autocorrelation and auto-covariance. 7

b) If people arrive at a book stall in accordance with a Poisson process with a mean rate of 3 per minute, find the probability that the interval between 2 consecutive arrivals is (a) more than 1 minute (b) between 1 minute and 2 minutes (c) 4 minutes or less. 7

16 a) A cooling system in a machine fails with mean rate of 1 per week following a Poisson process. Find the probability that 2 weeks have elapsed since last failure. If we have 5 extra cooling systems and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. 7

b) Consider the random process $X(t) = A\,cos(\omega t) + Bsin(\omega t)$ where A and B are independent random variables with zero mean and equal variance. Show that $X(t)$ is WSS. 7

Module -4

17 a) Solve the equation $xe^x - 2 = 0$ by Regula Falsi method. 7

b) Find the unique polynomial $P_3(x)$ of degree 3 or less, the graph of which passes through the points $(-1,3), (0,-4), (1,5)$ and $(2.-6)$. 7

161

18 a) Compare $\int_0^1 \frac{1}{1+x^2}$ by using (i)trapezoidal method (ii)Simpson's method with step size $h = 0.25$. Compare the results with exact value obtained by actual integration. 7

b) Using Newton's divided difference interpolating polynomial evaluate $y(8)$ from the following data. 7

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

Module -5

19 a) Solve the following system of equations using Gauss-Jacobi method starting with the initial approximation (0,0,0). 7

$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25.$

b) Fit a straight line $y = ax + b$ to the following data 7

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

20 a) Use Euler's method to solve $\frac{dy}{dx} = x + xy + y, y(0) = 1$. Compute y at $x = 0.15$ by taking $h = 0.05$. 7

b) Use Runge-Kutta method of fourth order to find y(0.1) from $\frac{dy}{dx} = \sqrt{x + y}$, y(0)=1 taking $h = 0.1$. 7

***

162