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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R,S) / S3 (WP) (R,S) / S1 (PT) (S,FE) Examination November 2024 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential Equations and Complex Analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Find the differential equation of all spheres of fixed radius having their centres in the xy-plane. (3)

2 Solve $\frac{\partial^2 z}{\partial x \partial y} = \frac{x}{y} + a$. (3)

3 Using d'Alembert's method, find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial deflection $f(x) = a(x - x^2)$. (3)

4 Derive the steady state solution of one dimensional heat equation. (3)

5 Test the continuity at $z = 0$ of $f(z) = \begin{cases} \frac{Re(z)}{1-|z|} & , z = 0 \\ 0 \quad , & z \neq 0 \end{cases}$ (3)

6 Show that an analytic function with constant real part is constant. (3)

7 Evaluate$\oint_{-\pi i}^{\pi i} \cos z \, dz$. (3)

8 Find the Maclaurin series of $\frac{1}{1+z^2}$. (3)

9 Find the zeros and their order of the function $(z) = (1 - z^4)^2$. (3)

10 Find the residue at poles for the function $(z) = \frac{sinhz}{z^4}$. (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Form the partial differential equations by eliminating the arbitrary functions from $z = y\,f(x) + x\,g(y)$. (7)

(b) Solve $(y + zx)p - (x + yz)q = x^2 - y^2$. (7)

12(a) Solve $q + xp = p^2$. (8)

(b) Using method of separation of variables, solve $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$. (6)

Module 2

13(a) Derive one dimensional wave equation. (7)

(b) A homogeneous rod of conducting material of length l has its ends kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature u(x,t). (7)

14(a) A tightly stretched homogeneous string of length l with its fixed ends at $x = 0$ and $x = l$ executes transverse vibrations. Motion starts with zero initial velocity by displacing the string into the form $f(x) = k(x^2 - x^3)$. Find the deflection u(x,t) at any time t. (8)

(b) Derive the solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ using variable seperable method. (6)

Module 3

15(a) Check whether $f(z) = iz\bar{z}$ is analytic. (7)

(b) Find the image of $x > 1, y > 0$ under the transformation $= \frac{1}{z}$. (7)

16(a) Show that $u = x^3 - 3xy^2 - 5y$ is harmonic. Also find the corresponding harmonic conjugate function. (7)

(b) Find the image of $|z| \le \frac{1}{2}, \frac{-\pi}{8} < Argz < \frac{\pi}{8}$ under the mapping $w = z^2$. (7)

Module 4

17(a) Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$ where C is the unit circle traversed counter clockwise. (6)

(b) Evaluate $\oint_C \frac{5z+7}{z^2+2z-3} dz$ where C is taken counterclockwise around the circle (8)

$i)\ |z - 2| = 2 \quad ii)\ |z + i| = 1$.

18(a) Integrate counterclockwise around the unit circle $\oint_C \frac{sin2z}{z^4} dz$. (7)

(b) Expand $f(z) = \frac{z-1}{z^2}$ as a Taylor series about $z_0 = 1$. (7)

Module 5

19(a) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ as a Laurent's series about $z = -2$ in $0 < |z + 2| < 1$ (5)

(b) Evaluate $\oint_0^{2\pi} \frac{cos2\theta}{5+4\,cos\theta} d\theta$ (9)

20(a) Using Residue theorem, evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle $|z - i| = 2$. (6)

(b) Using contour integration, evaluate $\oint_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$. (8)

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