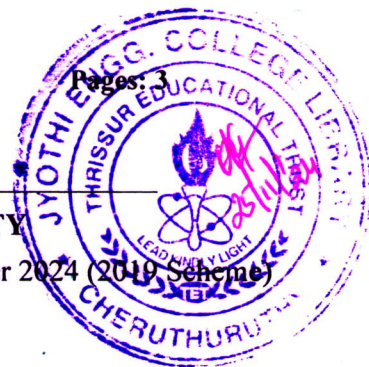


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Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R,S) / S3 (WP) (R,S) / S1 (PT) (S,FE) Examination November 2024 (2019 Scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks

Marks

- | | | |
|----|---|-----|
| 1 | Use truth table to prove that $p \wedge (p \rightarrow q) \rightarrow q$ is a tautology. | (3) |
| 2 | Give a proof by contradiction for the following
"for all integer n, if n^2 is odd then n is odd" | (3) |
| 3 | Find the number of arrangements of all the letters in "DATABASES" | (3) |
| 4 | In how many ways we can construct a secret code by assigning to each letter of the alphabet a different letter to represent it? | (3) |
| 5 | Define equivalence relation. Give an example. | (3) |
| 6 | Draw the Hasse diagram of $(D_{30}, /)$ where "/" is the divides operator. | (3) |
| 7 | Obtain a generating function for the sequence 1,2,3,4... | (3) |
| 8 | Solve the recurrence relation $a_n - na_{n-1} = 0$ where $a_0=1$. | (3) |
| 9 | Define group homomorphism. | (3) |
| 10 | Prove that the identity element of a group is unique. | (3) |

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

- | | | |
|-------|--|-----|
| 11(a) | Negate the statement "There exist real numbers x and y such that x and y are rational but $x + y$ is irrational". | (7) |
| (b) | Consider the open statement $p(x): x^2 \geq 1$. Discuss the truth value of the statement " $\forall x p(x)$ " where the universe consists of set of all positive integers. What if the universe is set of all real numbers? | (7) |
| 12(a) | Check the validity of the following argument. "If it does not rain or if there is no traffic dislocation, then the sports day will be held and cultural | (7) |

programmes will go on"; "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded" Therefore "It rained"

- (b) Define converse, inverse, and contrapositive and write same for the following statement. "If x is odd then x^2-1 is even" (7)

Module 2

- 13(a) If n is a positive integer prove that ${}^nC_2 + {}^{n-1}C_2$ is a perfect square. (7)

- (b) Determine the number of all non-negative integer solutions of the equation (7)

$$x_1 + x_2 + x_3 + x_4 = 7$$

- 14(a) Find the number of integers between 1 and 2000 that are not divisible by 2,3 or 5. (7)

- (b) Prove that in a room full of 367 people there are at least two people with the same birthday. (7)

Module 3

- 15(a) Check whether the relation R defined as " xRy if and only if $x-y$ is even" on set of all integers Z is an equivalence relation? If yes find the equivalence classes. (7)

- (b) Define a lattice. Prove that $P(S)$, the set of all subsets of a non-empty set S is a lattice under subset of ($\subseteq$) relation. (7)

- 16(a) Define least upper bound and greatest lower bound in a POSET. Consider the POSET $(R, \leq)$ and $A = \{x \in R / 0 \leq x \leq 1\}$. Find the LUB and GLB of A (7)

- (b) Define function. Explain domain, codomain and range of a function with the help of an example. (7)

Module 4

- 17(a) Solve the recurrence relation which generate the Fibonacci numbers 0,1,1,2,3,5... (7)

- (b) Solve $a_{n+1} = a_n + n$ $n > 1$ and $a_2 = 1$ (7)

- 18(a) Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200$, $a_0 = 3000$ and $a_1 = 3300$ (7)

- (b) Solve $a_n = 2a_{n-1} - 2a_{n-2}$ where $a_0 = 1$ & $a_1 = 2$. (7)

Module 5

- 19(a) Prove that $R^{2 \times 2}$, the set of all 2×2 real matrices is a group under matrix addition. Is it an Abelian group? (7)

- (b) Consider the groups $(\mathbb{Z}^+, +)$ and $(\mathbb{Z}^+, \times)$ where $\mathbb{Z}^+$ is the set of all positive integers. Check whether the function defined as $f(n)=3^n$ is a homomorphism of these two groups? Is it an isomorphism? (7)
- 20(a) If H and K are subgroups of a group G , prove that $H \cap K$ is a subgroup of G . (7)
- (b) Define order of an element in a group. Find the order of each element of the group $(\mathbb{Z}_4, +)$ where “+” is the addition modulo 4. (7)
