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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech (Hons.) Degree Examination June 2024 (2022 Admission)

Course Code: CST294

Course Name: COMPUTATIONAL FUNDAMENTALS FOR MACHINE LEARNING

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Consider the following system of equations and find solutions 3

$$x+y+z=2$$
$$x+3y+3z=0$$
$$x+3y+6z=3$$

2 Show that the linear mapping G defined as $G\big((x,y)\big) = (x+1, y+2)$ is not linear. 3

3 Let V be the vector space of polynomial with inner product given by $< f, f > = \int_0^1 f(t)f(t)dt$, where $f(t)$ = t+2. Find the norm for f. 3

4 Find the Eigenvalues and Eigenvectors of the 2 x 2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ 3

5 If $z = f(x+ct) + g(x-ct)$, prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$. 3

6 Find the second order Taylor series expansion for $f(x, y) = (x+y)^2$ about (0, 0). 3

7 A random variable follows binomial distribution with mean = 4 and variance = 3. Find the probability mass function. 3

8 Prove the Beta-Bernoulli Conjugacy. 3

9 Find all the basic solution of the following system of equations: 2x+y+4z =11, 3x+y+5z=14. 3

10 Explain the concept of Batch Gradient Descent 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Show that in the space R^3 the vectors x = (1, 1, 0), y = (0, 1, 2), and z = (3, 1, −4) are linearly dependent. 4

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b) Define linear combination, linear independence, linear dependence. Give examples. 10

12 a) Find the rank and inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix}$ 8

b) Find a basis for the vector space V spanned by vectors v1 = (1, 1, 0), v2 = (0, 1, 1), v3 = (2, 3, 1), and v4 = (1, 1, 1). 6

Module -2

13 a) Apply the Gram–Schmidt process to find an **orthonormal** basis for the space spanned by $v_1 = (1,2,2), v_2 = (-1,0,2), v_3 = (0,0,1)$. 8

b) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ 6

14 a) Find the singular value decomposition of $A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ 7

b) Diagonalize $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ and hence find A^{13}. 7

Module -3

15 a) A rectangular box open at the top is to have volume 32 cubic feet. Find its dimensions if the total surface area is minimum. 7

b) Find the directional derivative of $f(x, y) = xe^y - ye^x$ at the point P (0, 0) in the direction of 5i-2j. 7

16 a) Discuss the maxima and minima of $xy(a-x-y)$. 7

b) Show that $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y \sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and hence find its scalar potential. 7

Module -4

17 a) The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive? 9

b) Given a standard normal distribution, find the area under the curve that lies 5

(a) to the right of $z = 1.84$ and

(b) between $z = -1.97$ and $z = 0.86$.

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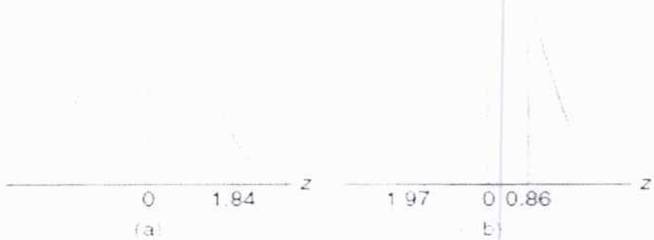

18 a) A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years. **7**

b) Define the probability density function and probability mass function. **7**

Module -5

19 a) Find the maxima, minima and saddle points for the following function **8**

$$f(x, y) = x^2 + y^2 - 2x + 4y + 8.$$

b) Determine whether the following functions are convex function or concave. Justify your answer. **6**

$$i. f(x) = 3x^2 + 7x - 9 \qquad ii. f(x) = -9x^2 - x - 1$$

20 a) Find minima using gradient descent method for the following function **8**

$$f(x) = x^2 - 2x + 2$$

b) For a rectangle whose perimeter is 20 m, find the dimensions that will maximize the area. **6**

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