Reg No.:______________ Name:____________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech (Minor) Degree Examination June 2024 (2022 Admission)

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Course Code: CST284

Course Name: Mathematics for Machine Learning

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1	Are the vectors **x= [2, 5, 1]** ^{T} and **y= [9,-3, 6]** ^{T} orthogonal? Justify your answer.	3
2	Let X be a continuous random variable with pdf $f(x) = e^{-X}$, x>=0. Determine the pdf of $Y=X^2$	3
3	With usual operation of scalar multiplication, but with addition on R given by **x#y= (2x+2y).** Check whether it forms a vector sapce.	3
4	Develop the binomial distribution for which mean is 5 and variance is 4.	3
5	Compute the angle between $x=\begin{bmatrix}1\\2\end{bmatrix}$ and $y=\begin{bmatrix}-1\\-1\end{bmatrix}$ using a. $\langle x,y\rangle = x^T y$ b. . $\langle x,y\rangle = x^T By$, $B=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$	3
6	Find a binomial distribution of mean 4 and variance 2. Find the probabilities of getting i) At least 2 success ii) At most 2 success iii) Find the probability of 5<=x<=7	3
7	State the Central limit theorem.	3
8	Consider a polynomial $f(x) = \sin(x) + \cos(x) \in C\infty$.Seek Taylor series expansion of f at x0 = 0.	3
9	If x is a uniform distribution in (3,-3). Find P(x<1), P (|x|>2), P (|x-2|<2).	3
10	Consider the univariate function: $f(x) = x^3 + 6x^2 -3x-5$. Find its stationery points and indicate whether they are maximum, minimum or saddle points.	3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	A set of n linearly independent vector in R^n forms a Basis. Does the set of vectors **(2,4,-3), (0,1,1), (0,1,-1)** form a basis for R^3.Explain your reasons	8
	b)	Find for what value of a, is the system: **-2x+4y-2z-w+4v=-3; 4x-8y+3z-3w+v=2; x-2y+z-w+v=0; x-2y+0z-3w+4v=a,** consistent. Hence solve the same	6
12	a)	Find the dimension and the basis of the vector spaces spanned by the vectors (1,1,-2,0,-1),(1,2,0,-4,1),(0,1,3,-3,2),(2,3,0,-2,0)	8
	b)	What is a linear transformation. Explain terms kernal, range and null space.	6

Module -2

13	a)	Find the characteristic equation, and Eigen spaces corresponding to each Eigen value of the following matrix **2 0 4** **0 3 0** **0 1 2**	6
	b)	Factorize the given matrix A into $U\sum V^T$ using SVD A= $\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}$	8
14	a)	Use Gram-Schmidt Process to find an orthogonal basis from the ordered basis B={b1,b2},where b1=$\begin{bmatrix} 2 \\ 0 \end{bmatrix}$and b2=$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	6
	b)	IDiagonalizable the symmetric matrix $\mathbf{A} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	8

Module -3

15	a)	Find the second order Taylor series expansion for **f(x,y)=**$e^{-(x2+y2)}$**cos(xy)** about **(0,0)**	6
	b)	Compute the gradient of the Rectified Linear Unit(ReLU) function **ReLU(z)= max(0,z)**	8
16	a)	Consider the following functions: $f_1(x)= \sin(x1)\cos(x2),\ x \in R^2$ $f_2(x,y)= x^T y,\ x,y \in R^n$ $f_3(x)= xx^T,\ x \in R^n$ a. What are the dimensions of $\partial fi/\partial x$? b. Compute the Jacobians.	7

b) Write down the Logistic Sigmoid function. Find its derivative using chain Rule .Give one specific application of the function in machine learning 7

Module -4

17 a) A coin for which P (heads) = p is tossed until two successive tails are obtained. Find the probability that the experiment in completed on the n^{th} toss 6

b) Suppose that one has written a computer program that sometimes compiles and sometimes not. You decided to model the apparent stochasticity (success vs. no success) x of the compiler using a Bernoulli distribution with parameter **μ:** $P(x|\mu) = \mu^x (1-\mu)^{1-x}$, x€ {0, 1}. Choose a conjugate prior for the Bernoulli likelihood and compute the posterior distribution $p(\mu|x_1 \ldots x_N)$. 8

18 a) In a competitive examination, 5000 students have appeared .The average score was 62 and standard deviation of scores 12. If there are only 100 vacancies, find the cut off marks to be scored for selection, modelling the problem using suitable Probability distribution 8

b) Two dice are rolled. 6

A= sum of two dice equals 3

B= sum of two dice equals 7

C= at least one of the dice shows a 1

i) Evaluate P (A|C)

ii) Evaluate P (B|C)

iii) Are A and C independent? What about B and C

Module -5

19 a) Consider the function $f(x) = 1/2\, x^T A x + b^T x + c$;

Where A is strictly positive definite, which means that it is invertible. Derive the convex conjugate of f(x). 7

b) Consider the negative entropy of $x \in R^D$, $f(x) = \sum x_d \log x_d$, d=1, … D

Derive the convex conjugate function f*(s), by assuming the standard dot product. 7

20 a) Consider the following convex optimization problem 8

Minimize $x^2/2 + x + 4y^2 - 2y$, subject to the constraint $x + y >= 4$, x, y>=1.

Derive an explicit form of the Lagrangian dual problem.

b) Is the function $f(x, y) = 2x^2 + y^2 + 6xy - x + 3y - 7$ convex, concave or neither? Justify you answer. 6

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