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Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S,FE) / S1 (PT) (S,FE) Examination June 2024 (2015 Scheme)

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) Show that $u = SinxCoshy$ is harmonic and hence find its harmonic conjugate. (8)

b) Test the analyticity of $f(z) = z^3$. Find $f^1(z)$. (7)

2 a) Find the images of $\frac{-\pi}{2} < x < \frac{\pi}{2}$, $1 < y < 2$ under the mapping $w = Sinz$. (8)

b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ into the points $w = 0, i, \infty$ respectively. (7)

3 a) Show that the derivative of $f(z) = \bar{z}$ does not exist anywhere. (7)

b) Describe the transformation $w = e^z$. (8)

PART B

Answer any two full questions, each carries 15 marks

4 a) Evaluate $\int_C z^2 dz$ where C is the straight line joining the origin (0.0) and the point (2,1) on the complex plane. (7)

b) Find all Laurent's series expansion of $f(z) = \frac{z}{(z-1)(z-3)}$ about z=1 valid for (i) $0 < |z-1| < 2$ (ii) $|z-1| > 2$ (8)

5 a) Evaluate $\oint_C \frac{dz}{z^2+4}$ where C is the ellipse $4x^2 + (y-2)^2 = 4$ using residue theorem. (7)

b) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$. (8)

6 a) Evaluate $\oint_C \frac{3z^2+z-1}{(z-1)^2(z-3)} dz$ where C is (i) the circle $|z| = 2$, (ii) the circle $|z| = \frac{1}{2}$ by Cauchy's integral formula. (7)

b) Show that $\int_0^{\pi} \frac{d\theta}{(2+Cos\theta)^2} = \frac{2\pi}{3\sqrt{3}}$. (8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Solve using Gauss elimination method (6)

$$2x + 3y + 4z = 11 \; ; x + 5y + 7z = 15 \; ; \; 3x + 11y + 13z = 25$$

b) Check whether the following vectors are linearly independent or not. (4)

$$(1, 2, 3), (3, -2, 1) \; and \; (1, -6, -5)$$

c) Reduce the following quadratic form into its canonical form. (10)

$$Q(x_1, x_2, x_3) = 2x_1x_2 + 2x_1x_3 - 2x_2x_3$$

8 a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. Is this matrix orthogonal? (10)

b) Find a basis for the null space of $\begin{bmatrix} 2 & -2 & 0 \\ 0 & 4 & 8 \\ 2 & 0 & 4 \end{bmatrix}$ (5)

c) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 2 & 9 & 12 \\ 1 & 17 & 21 \end{bmatrix}$ (5)

9 a) Determine the values of λ, k for which the following system of equations (10)

$$x + y + z = 6\,; x + 2y + 3z = 10\,; x + 2y + kz = \mu$$

possesses (i) no solution (ii) unique solution (iii) infinite solution

b) Diagonalise the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ (10)

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