C 15266 (Pages 2) Name.........................

Reg. No.........................

FOURTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2010

CH 2K 401—ENGINEERING MATHEMATICS—IV

(Common to AI/CE/EE/IC/ME/EC/PE)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Prove that the function $f(z) = z - \overline{z}$ is nowhere differentiable.

(b) Find the image of the region $1 \leq x \leq 2$ under the mapping $W = e^z$.

(c) Evaluate $\int_C \frac{z}{z^2+1} dz$, where C is $|z + i| = 1$.

(d) Determine the poles and residue at each pole of the function $f(z) = \cot z$.

(e) Prove that $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_n(x)$.

(f) Express $4x^3 - 3x + 8$ in terms of Legendre polynomials.

(g) Classify the equation $e^x u_{xx} + e^y u_{yy} = u$ and find its characteristic equation.

(h) Using method of seperation of variables, solve $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$.

(8 × 5 = 40 marks)

II. (a) (i) If $f(z) + u + iv$ is an analytic function, then show that u and v are both harmonic functions.

(ii) Determine the analytic function $f(z) = u + iv$, where $v = \frac{-y}{x^2 + y^2}$.

Or

(b) Find the bilinear transformation which maps the points :

$$z_1 = \infty, z_2 = i, z_3 = 0 \text{ into } w_1 = 0, w_2 = i, w_3 = \infty.$$

Find the invariant point of this transformation.

Turn over

III. (a) Expand the function $f(z) = \dfrac{1}{(1+z^2)(2+z)}$, where :

1 $|z| < 1$, 2 $1 < |z| < 2$.

3 $|z| > 2$.

Or

(b) Evaluate $\int_0^{2\pi} \dfrac{\cos\theta}{3+\sin\theta} d\theta$.

IV. (a) Solve the equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$$

in power series.

Or

(b) (i) Show that :

$$(n+1)\, p_{n+1}(x) = (2n+1)\, x\, p_n(x) - n P_{n-1}(x).$$

(ii) Show that $p'_{2n}(0) = 0$.

V. (a) A string is stretched and fixed between two points (0, 0) and $(l, 0)$. Motion is initiated by displacing the string in the form $u = \lambda \sin\left(\dfrac{\lambda x}{l}\right)$ and released from rest at time $t = 0$. Find the displacement of any point on the string at any time l.

Or

(b) Solve $u_t = u_{xx}, 0 < x < 1, t > 0$ subject to the conditions $u(0, t) = 1$, $u(1, t) = 1$, $t > 0$ $u(x, 0) = 1 + \sin \pi x, 0 < x < 1$.

(4 × 15 = 60 marks)