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Reg No.:_______________ Name:_______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree (S, FE) Examination June 2024 (2019 Scheme)

Course Code: MAT 101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1 Find the rank of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$. (3)

2 Find the sum and product of eigen values of $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ without finding the characteristic equation. (3)

3 Find the slope of the sphere $x^2 + y^2 + z^2 = 14$ in the y direction at $(1,2,3)$ (3)

4 Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$, where $z = 10x^5y^3 + 5x + 2y$ (3)

5 Find the area of the region bounded by $y = x^2$ and $y = x$. (3)

6 Evaluate $\int_2^4 \int_1^3 (40 - 2xy)\,dxdy$. (3)

7 Test the convergence of the series $\sum_{k=1}^{\infty} \frac{99^k}{k!}$ (3)

8 Test the convergence of the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ (3)

9 Find the Maclaurin series for the function $f(x) = xe^x$ (3)

10 Write Binomial series for $(1 + x^2)^3$ (3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following system of equations using Gauss elimination method (7)

$y - 3z = -1$

$x + z = 1$

$3x + y = 2$

$x + y - 2z = 0$

b Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$. (7)

12 a Find the matrix of the transformation that diagonalise the matrix $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$. (7)

Also write the diagonal matrix.

b Find the value of α for which the system of equation is consistent. (7)

$x+y+z=1$

$x+2y+3z=\alpha$

$x+5y+9z=\alpha^2$

MODULE 2

13 a Find the local linear approximation L of $f(x,y,z)=\log(x+yz)$ at the point $(2,1,-1)$. (7)

b If $w=f(P,Q,R)$ where $P=2x-3y,\ Q=3y-4z,\ R=4z-2x$, then prove that $\frac{1}{2}\frac{\partial w}{\partial x}+\frac{1}{3}\frac{\partial w}{\partial y}+\frac{1}{4}\frac{\partial w}{\partial z}=0$ (7)

14 a Locate all relative extrema and saddle points of $x^3+y^3-3xy=0$. (7)

b Find the differential dw of the functions. (7)

i) $w=\frac{xyz}{x+y+z}$

ii) $w=e^{xy}$

MODULE 3

15 a Evaluate $\iint_R \frac{1}{1+x^2+y^2}\,dA$ where R is the sector in the first quadrant bounded by $y=0,\ y=x,\ x^2+y^2=9$. (7)

b Evaluate the integral $\int_0^4\int_y^4 \frac{x}{x^2+y^2}\,dxdy$ by reversing the order of integration. (7)

16 a Use triple integral to find the volume of the solid within the cylinder $x^2+y^2=9$ and between the planes $z=1$ and $x+z=5$. (7)

b Find the center of gravity of a triangular lamina with vertices (0,0), (0,1) and (1,0) and density function $\rho(x,y)=xy$ and mass $=\frac{1}{24}$. (7)

MODULE 4

17 a A ball is dropped from a height of 10m. Each time it strikes the ground it (7)

bounces vertically to a height that is $\frac{2}{3}$ of the preceding height. Find the total distance travelled by the ball, if it is assumed to bounce infinitely often.

b Check the convergence the following series (7)

i) $\sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)(2n+3)}$

ii) $\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1}\right)^{n^2}$

18 a Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$ is conditionally convergent. (7)

b Check the convergence of the series $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \cdots$ (7)

MODULE 5

19 a Find the Taylor series expansion of $f(x) = xsinx$ about the point $x = \frac{\pi}{2}$ (7)

b Find the Fourier series representation of $f(x) = x^2$ in $[-\pi, \pi]$ and deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$ (7)

20 a Find the half range Fourier cosine series of $f(x) = cosx$ in $0 \leq x \leq \frac{\pi}{2}$ (7)

b Find the half range Fourier sine series of $f(x) = e^x$ in $(0,1)$ (7)

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