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Reg No.:_____ Name:_____

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

Fourth Semester B.Tech Degree (R, S) Examination May 2024 (2019 Scheme)

**Course Code: MAT256**

**Course Name: PROBABILITY AND STATISTICAL MODELLING**

Max. Marks: 100 Duration: 3 Hours

**PART A**

***(Answer all questions; each question carries 3 marks)*** Marks

| | | | |
|---|---|---|---|
| 1 | | Six dice are thrown 729 times. How many times do you expect at least three dice to show 1 or 2? | 3 |
| 2 | | If X is a Poisson variable such that P(X=1) =3/10 and P(X=2) =1/5, find the mean $\lambda$ | 3 |
| 3 | | Define Normal Distribution and Standard Normal Distribution | 3 |
| 4 | | A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find P(X < 0) | 3 |
| 5 | | Discuss the difference between F-distribution and Chi-square distribution. | 3 |
| 6 | | Define Standard Error. Explain with an Example. | 3 |
| 7 | | Define Null Hypothesis, Alternative Hypothesis and Level of Significance. | 3 |
| 8 | | A sample of 20 items has mean 42 and SD 5. Test whether the sample is from a population with mean 45? ( 5% level of significance) | 3 |
| 9 | | Explain Spearman's rank correlation. | 3 |
| 10 | | Explain partial and multiple correlation. | 3 |

**PART B**

***(Answer one full question from each module, each question carries 14 marks)***

**Module -1**

| | | | |
|---|---|---|---|
| 11 | a) | Out of 2000 families with 4 children each, how many would you expect to have (i) at least one boy (ii) at most one boy | 7 |
| | b) | The joint pdf of X and Y is given by f(x, y) = (2x + y)/27; x = 0, 1, 2 and y = 0, 1, 2. Find the marginal distributions of X and Y. Find also the means of X and Y. | 7 |

12 a) If mean and variance of a Binomial Distribution are 30 and 25 respectively, find P(X=3). 7

b) A random Variable follows a Poisson distribution with mean 1. Calculate the probability that (i) X = 0 (ii) X =1 (iii) X > 2 7

**Module -2**

13 a) Buses arrive at a specified stop at 15 minutes interval starting at 8 am. If a passenger arrives at the stop at a random time that is uniformly distributed between 8.00 and 8.30 hours, find the probability that the passenger waits for (i) less than 6 minutes for the bus (ii) at least 12 minutes for the bus 7

b) If a continuous random variable has the probability distribution function

$f(x) = ke^{-3x}, x > 0$

$= 0, otherwise.$ 7

Then find (i) value of k (ii) P(0 < X < 2)

14 a) A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3? 7

b) The joint PDF of (X,Y ) is given by f(x, y) = kxy ; 0<x<4; 1<y<4

$= 0$ ; elsewhere 7

Find value of k. Determine marginal pdf of X and Y. Evaluate P[X>2, Y>3]

Check whether X and Yare independent?

**Module -3**

15 a) The mean age at death of 64 men engaged in an occupation is 52.4 years with standard deviation 10.2 years. What are the 99% confidence limits for the mean age of all men in that occupation? 7

b) Determine the size of the sample for estimating the true weight of the containers for the universe on the basis of the following information. 7

(1) The variance of weight is 4 ounces.

(2) Estimate should be within 0.8 ounces of the true average weight with 99% probability.

16 a) A distribution with unknown mean has variance 1.5. Use Central Limit Theorem to find, how large a sample should be taken from the distribution in order that the 7

probability that the sample mean will be with in the 0.5 of the population mean is 0.95.

b) The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem, find the probability that the average life time of 60 lights exceeds 1250 7

**Module -4**

17 a) A sample of 25 items were taken from a population with standard deviation 10 and the sample mean is found to be 65. Can it be regarded as a normal population with mean = 60. 7

b) A sample of 50 lenses used in eyeglasses yields a sample mean thickness of 3.05 mm and a sample standard deviation of .34 mm. The desired true average thickness of such lenses is 3.20 mm. Does the data strongly suggest that the true average thickness of such lenses is something other than what is desired? Test using $\alpha$=0.05 7

18 a) The recommended daily dietary allowance for zinc among males older than age 50 years is 15 mg/day. The article "Nutrient Intakes and Dietary Patterns of Older Americans: A National Study" reports the following summary data on intake for a sample of males age 65–74 years: n =115, $\bar{x}$ =11.3, and s = 6.43. Does this data indicate that average daily zinc intake in the population of all male's ages 65–74 falls below the recommended allowance? 7

b) A trucking company wishes to test the average life of each of the four brands of tyres. The company uses all brands on randomly selected trucks. The records showing the lives (thousands of miles) of tyres are given below. Test the hypothesis that the average life for each brand of tyres is the same. ( $\alpha$ = 0.01) 7

| Brand 1 | Brand 2 | Brand 3 | Brand 4 |
|---|---|---|---|
| 21 | 19 | 24 | 15 |
| 22 | 19 | 19 | 19 |
| 19 | 17 | 20 | 16 |
| 16 | 24 | 19 | 15 |
| | 16 | 16 | |

## Module -5

19 a) Find the correlation coefficient for the following data 7

| X | 1 | 2 | 3 | 4 | 6 | 8 |
|---|---|---|---|---|---|---|
| Y | 24 | 3 | 26 | 4 | 5 | 6 |

b) Suppose that in a certain chemical process the reaction time $y$ (hr) is related to the temperature (°F) in the chamber in which the reaction takes place according to the simple linear regression model with equation $y = 5.00 - 0.01x$. 7

(i) What is the expected change in reaction time for a 1°F increase and 10°F increase in temperature?

(ii) What is the expected reaction time when temperature is 200°F and 250°F?

20 a) Write the regression line of X on Y from the following data

| X | 60 | 72 | 69 | 58 | 73 | 82 | 85 | 76 | 82 | 65 |
|---|---|---|---|---|---|---|---|---|---|---|
| Y | 65 | 68 | 75 | 62 | 75 | 83 | 87 | 78 | 85 | 66 |

7

b) The flow rate $y$ (m'/min) in a device used for air-quality measurement depends on the pressure drop x (inches of water) across the device's filter. Suppose that for x values between 5 and 20, the two variables are related according to the simple linear regression model with true regression line 7

$$y = -0.12 + 0.095x$$

(i) What is the expected change in flow rate associated with a 1 inch increase in pressure drop? Explain.

(ii) What change in flow rate can be expected when pressure drop decreases by 5 inches?

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