Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S,FE) / S1 (PT)(S) June 2024 (2019 Scheme)/S3 (WP)(R) December 2023 Examination

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Construct a truth table for the compound statement $q \leftrightarrow (\neg p \vee \neg q)$ where p and q are primitive statements. Check whether it is a tautology.	(3)
2	If p, q and r are primitive statements, write the converse, inverse and contrapositive of $p \rightarrow (q \wedge r)$.	(3)
3	What are the number of possible arrangements of the letters in the word "DATABASES".	(3)
4	Given a group of 100 people, at minimum how many people were born in the same month ?	(3)
5	Give an example of a relation R on $\mathbb{Z}$ where R is irreflexive, transitive but not symmetric. ($\mathbb{Z}$ -is the set of integers)	(3)
6	Draw the Hasse diagram of $(D_{20}, \mid)$. ($\mid$ is the divides relation)	(3)
7	Find the sequence generated by the generating function $\frac{1-x^{n+1}}{1-x}$.	(3)
8	Determine the constant term in the expansion of $\left(4x^3 - \frac{5}{x}\right)^{16}$.	(3)
9	Let E be the set of all positive integers 2,4,6, ... Define $f: Z^+ \rightarrow E$ by $f(n) = 2n$. Is f a semigroup homomorphism from $(Z^+, \cdot)$ into $(E, \cdot)$.	(3)
10	Prove that every cyclic group is abelian.	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a)	Prove that $\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$	(6)
(b)	Find the truth values of inverse, converse and contrapositive of the statement 'If the magnitude of a real number is greater than 3, then the number itself is greater than 3' where the universe is the set of all real numbers.	(8)

12(a) Check whether the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$ are logically equivalent. (6)

(b) Establish the validity of the argument (8)

$$p \rightarrow q$$
$$q \rightarrow (r \wedge s)$$
$$\neg r \vee (\neg t \vee u)$$
$$p \wedge t$$

....

$$\therefore u$$

Module 2

13(a) Determine the coefficient of $w^3x^2yz^2$ in the expansion of $(2w - x + 3y - 2z)^8$ (6)

(b) Determine the number of integral solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where i) $x_i \geq 0, \quad 1 \leq i \leq 4$ ii) $x_1, x_2 \geq 5, x_3, x_4 \geq 7$ (8)

14(a) A certain ice cream store has 31 flavours of ice cream available. In how many ways can we order a dozen ice cream cones if (i) we do not want the same flavour more than once? (ii) a flavour may be ordered as many as 12 times (iii) a flavour may be ordered no more than 11 times. (6)

(b) Find the number of integers between 1 and 10000 inclusive, which are divisible by 5, 6 or 8. (8)

Module 3

15(a) If R is a relation on the set $\mathbb{Z}$ of all integers defined by $R = \{(x, y): x \in Z, y \in Z, x - y \text{ is divisible by } 3\}$. Prove that R is an equivalence relation. Describe the distinct equivalence classes of R. (6)

(b) Show that $(D_{42}, |)$ is a lattice. Find the complements of each element and check whether it is a complemented lattice. (8)

16(a) If f, g and h are functions on the set of all real numbers, $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ then find (6)

(i)$(f \circ g) \circ h$ (ii) $h \circ g \circ f$ (iii) $f \circ (g \circ h)$

(b) Let $A = \{1, 2, 3, 5, 30\}$ i) Show that $(A, |)$ is a lattice

ii) Prove that "$\cdot$" is not distributive over "$+$" in this lattice by identifying elements a, b, c in A for which $a \cdot (b + c) \neq (a \cdot b) + (a \cdot c)$. (8)

iii) Prove that " + " is not distributive over " · " in this lattice by identifying elements a, b, c in A for which $a + (b \cdot c) \neq (a + b) \cdot (a + c)$..

Module 4

17(a) Find the coefficient of x^{20} in $f(x) = (x^2 + x^3 + x^4 + x^5 + x^6)^5$. (6)

(b) Solve the recurrence relation (8)

$$a_n - 3a_{n-1} - 4a_{n-2} = 4^n; n \geq 2, a_0 = 3, a_1 = \frac{26}{5}$$

18(a) Solve the recurrence relation (6)

$$a_n = 2(a_{n-1} - a_{n-2}); n \geq 2, a_0 = 1, a_1 = 2$$

(b) Solve the recurrence relation (8)

$$a_{n+2} - 8a_{n+1} + 19a_n = 8(5)^n + 6(4)^n; n \geq 2, a_0 = 12, a_1 = 5$$

Module 5

19(a) Show that any group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. (6)

(b) If H and K are subgroups of group G, where e is the identity of G. Prove that $H \cap K$ is a subgroup of G. Also, if $|H| = 10$ and $|K| = 21$ then $H \cap K = \{e\}$. (8)

20(a) Show that (6)

(i)The order of any element of a finite group is a divisor of the order of the group.

(ii)If G is a finite group of order n then $a^n = e$ for any $a \in G$.

(b) If $A = \{1,2,3\}$. List all permutations on A and prove that it is a group. (8)

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