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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S,FE) / S2 (PT) (S,FE) Examination May 2024 (2015 Scheme)

Course Code: MA204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A

Answer any two full questions, each carries 15 marks

1 a) Let X be a discrete random variable whose cumulative distribution function is F(x)= 0 if x<-2 — 7

$=\frac{1}{8}$ if $-2\leq x<0$

$=\frac{1}{4}$ if $0\leq x<2$

$=\frac{1}{2}$ if $2\leq x<4$

$=1$ if $x>4$.

i) Find the probability mass function of X

ii) Find $P(X\leq 1)$, $P(-1<X\leq 1)$

b) A pair of fair dice is thrown 5 times. If getting a doublet is considered success find the probability of getting i) at least 2 success ii) atmost 2 success iii) exactly 2 failures. — 8

2 a) Derive the mean and variance of uniform distribution — 7

b) In a class quiz, 8% of students obtained marks below 25 and 90% of the students got marks below 85. Assuming marks are normally distributed find mean and variance. — 8

3 a) The time required (in hours) to repair a machine is exponentially distributed with parameter $\lambda=\frac{1}{3}$. — 7

i) What is the probability that the repair time exceeds 3 hours?

ii) What is the conditional probability that repair time takes at least 8 hours given that its duration exceeds 2 hours?

b) In a component manufacturing industry there is a small probability of $\frac{1}{500}$ of any component to be defective. The components are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i)at least 1 defective ii)at most 1 defective ,in a consignment of 1000 packets 8

PART B

Answer any two full questions, each carries 15 marks

4 a) The joint probability distribution of X and Y is given by f(x,y)= $\frac{x+2y}{54}$ for x=1,2,3 and y= 1,2,3. 7

i) Find the marginal distributions of X and Y.

ii)Find the means of X and Y.

b) The life time of a certain type of electric bulbs may be considered to follow exponential distribution with mean 50 hrs. Use central limit theorem to find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hrs of burning time. 8

5 a) Show that the random process X(t) = ACos(ωt+θ) is a WSS if A and ω are constants and θ is a uniformly distributed random variable in (0,2π). 7

b) The autocorrelation function for a stationary process X(t) is given by 8

$R_{XX}(\tau) = 9+2e^{-|\tau|}$. Find the mean value of the random variable $Y=\int_0^2 X(t)dt$.

6 a) The joint density function of two continuous random variables X, Y is given by 7

$$f(x,y) = \begin{cases} K(1-x-y), & 0<x<\frac{1}{2}; 0<y<\frac{1}{2} \\ 0, & otherwise \end{cases}$$ Find (i) the value of K

(ii) the marginal distributions of X, Y (iii) check whether X, Y are independent.

b) The power spectral density of a WSS process is $\frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the autocorrelation function and power of the process. 8

PART C

Answer any two full questions, each carries 20 marks

7 a) Prove that the sum of two independent Poisson processes is again a Poisson process 6

b) A radioactive source emits particles at the rate of 6 per minutes in accordance with 6

Poisson process. Each particle emitted has a probability of $\frac{1}{3}$ being recorded. Find the probability that atleast 5 particles are recorded in 5 minutes.

c) There are 2 white balls in bag A and 3 red balls in bag B. At each step of the process a ball is selected from each bag and the 2 balls selected are interchanged. What is the probability that there are 2 red balls in bag A after 3 steps? In the long run what is the probability that there are two red balls in bag A? 8

8 a) Find the $\sqrt[3]{7}$ (cube root of 7) correct to 4 decimal places by Newton's method 6

b) The following table gives the values of cos(θ) where θ is in degrees 8

θ	5	10	15	20	25	30
Cos(θ)	0.9962	0.9848	0.9659	0.9397	0.9063	0.8660

Using suitable Newton's interpolation formulae evaluate i) Cos(9°) ii) Cos(28°)

c) Evaluate $\int_0^2 xe^x dx$ using Simpson's 1/3rd rule with 8 subintervals. 6

9 a) Solve the initial value problem $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y(0)=1and hence find y(0.1) by Euler method by taking h = 0.002 5

b) Using Runge Kutta method of order 4 compute y(0.1) from the equation $\frac{dy}{dx}=3x+\frac{y}{2}$,given y(0) =1 taking h = 0.1 5

c) In a tropical city suppose that the probability of a sunny day (state 0) following a rainy day (state 1) is 0.5 and that the probability of a rainy day following a sunny day is 0.3.We are given that the New Year day is a sunny day. 10

i) Find the transition probability matrix of the Markov chain.

ii) Find the probability that January 3rd is a Sunny day

iii) Find the probability that January 4rh is a rainy day

iv)In the long run what will be the proportion of sunny days and rainy days in the given city.

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