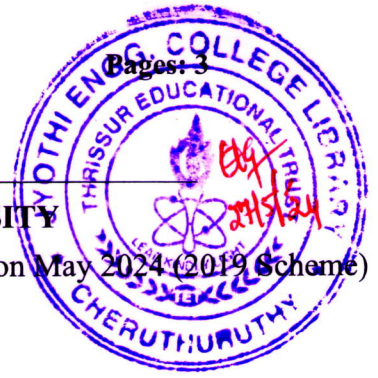


Reg No.: _____

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (R,S) / S4 (PT) (R,S) / S2 (PT) (S,FE) / S4 (WP) (R) Examination May 2024 (2019 Scheme)

**Course Code: MAT204****Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS**

Max. Marks: 100

Duration: 3 Hours

PART A*(Answer all questions; each question carries 3 marks)*

Marks

- 1 Determine the binomial distribution for which mean is 4 and variance is 3. 3
- 2 X follows a Poisson distribution with mean 6. Find $P(X = 1)$, Variance (X). 3
- 3 If X has a Uniform Distribution in $(-3,3)$, find $P(|X - 2| < 2)$. 3
- 4 The joint probability density of a two-dimensional random variable is

$$f(x,y) = \begin{cases} \frac{xy}{96}, & 0 < x < 4, \quad 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$
Find $P(1 < X < 2, 2 < Y < 3)$. 3
- 5 Define stationary random process. Define two types of stationary random process. 3
- 6 A random process $X(t) = A \cos \omega t, t \geq 0$ where ω is a constant and A is uniformly distributed in $(0,3)$. Determine $E[X(t)]$. 3
- 7 Use trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five subintervals. 3
- 8 Using Newton Raphson method find the positive root of the equation $x^3 - 24 = 0$ that lies between 2 and 3 correct to three decimal places. 3
- 9 Write the normal equations for fitting the curve $y = a + bx^2$. 3
- 10 Given $\frac{dy}{dx} = 1 - y, y(0) = 0$. Use Euler's method with $h = 0.1$, to compute the value of $y(0.2)$. 3

PART B*(Answer one full question from each module, each question carries 14 marks)***Module -1**

- 11 a) Let X be a random variable taking values $-1, 0$ and 1 such that $P(X = -1) = 2P(X = 0) = P(X = 1)$. Find (i) the pmf of X (ii) the mean of $(2X - 5)$. 7
- b) It is known that 2% of the bolts produced by a company are defective. The bolts are supplied in boxes of 200 bolts. What is the probability that a randomly

chosen box contains not more than 5 defective bolts? In a consignment of 1000 such boxes how many can be expected to have more than 5 defective bolts? (Use Poisson distribution) 7

- 12 a) A random variable X has the following probability distribution:

x	-2	-1	0	1	2	3
$f(x)$	0.1	$15k^2$	0.2	$2k$	0.3	$3k$

- (i) Find the value of k (ii) the mean and variance of X . 7
- b) Derive the mean and variance of Poisson distribution. 7

Module -2

- 13 a) A random variable X has the following probability density function $f(x) = kx(2 - x), 0 < x < 2$. Find (i) the value of k (ii) the mean (iii) variance 7
- b) The marks obtained by students in an intelligence test follow normal distribution with mean 45 and standard deviation 25. Find the percentage of students who scored marks (i) more than 80 (ii) between 30 and 70 (iii) below 35. 7
- 14 a) If the mileage, which a car owner gets with a certain kind of radial tyre, is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tyres will last (i) at least 20,000 km (ii) at most 30,000 km. 7
- b) A distribution with unknown mean μ has variance 1.5. Use Central Limit Theorem to find, how large a sample should be taken from the distribution in order that the probability that the sample mean will be in the 0.5 of the population mean is 0.95. 7

Module -3

- 15 a) Let $\{X(t) = A \cos \omega t + B \sin \omega t, t > 0\}$ be a random process where A and B are independent random variable following normal distribution with mean 0 and variance 4. Check whether $\{X(t)\}$ is WSS. 7
- b) Find the power spectral density function of the WSS process whose autocorrelation function is $A^2 e^{-2\alpha|\tau|}$ where A and α are constants. 7
- 16 a) If $\{X(t)\}$ is a random process with mean 3 and $R(t_1, t_2) = 9 + 4e^{-\left(\frac{|t_1 - t_2|}{5}\right)}$. Find (i) $V(X(5))$ (ii) $V(X(8))$ (iii) $Cov(X(5), X(8))$. 7
- b) Show that the sum of two independent Poisson processes is also a Poisson process. 7

Module -4

- 17 a) Find the root of the equation $\cos x - xe^x = 0$ that lies between 0 and 1, using Regula- Falsi method, correct to four decimal places. 7

- b) Calculate $y(0.015)$ using Newton's forward interpolation formula

x	0.01	0.02	0.03	0.04	0.05
y	1.2	2.5	3.6	4.6	5.3

- 18 a) Using Lagrange's interpolation, find the value of $y(8)$, given $y(0) = 18, y(1) = 42, y(7) = 57, y(9) = 90$. 7

- b) Evaluate $\int_0^{0.6} \frac{dx}{\sqrt{1+x}}$ with $n = 6$. 7

Module -5

- 19 a) Using Gauss-Seidel iteration method, find an approximate solution to the following system of equations correct to 4 decimal places. 7

$$8x - 3y + 2z = 20, \quad 4x + 11y - z = 33, \quad 6x + 3y + 12z = 36$$

- b) Using Runge Kutta method of order 4, find $y(0.1)$ given, $y' = x + y, y(0) = 1$ by taking $h = 0.1$. 7

- 20 a) Fit a second-degree parabola of the form $y = a + bx + cx^2$ to the following data 7

x	0	1	2	3	4
y	1.2	1.7	2.1	2.8	5.9

- b) Solve $\frac{dy}{dx} = x^2(1 + y)$ for $x = 1.4$ using Adams-Moulton Method, given $y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548$ and $y(1.3) = 1.979$. 7
