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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S,FE) S2 (S,FE) Examination May 2024 (2015 Scheme)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 5 Marks

			Marks
1	a)	Determine whether the series $\sum_{k=1}^{\infty}(\frac{-2}{3})^{k+1}$ converges. If so find its sum.	(2)
	b)	Use alternating series test, determine whether the series $\sum_{k=1}^{\infty}(-1)^{k+1}e^{-k}$ converge or not.	(3)
2	a)	Find the slope of the surface $z = xe^{-y} + 5y$ in the x-direction at the point (4, 0).	(2)
	b)	Show that the function $f(x,y) = e^{x}siny + e^{y}sinx$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$.	(3)
3	a)	Find the gradient of $f(x,y) = x^2e^y$ at the point (-2, 0)	(2)
	b)	Find the velocity and speed of a particle moving along the curve $\vec{r}(t) = e^t\hat{\imath} + e^{-t}\hat{\jmath}$ at time $t = 0$.	(3)
4	a)	Evaluate $\int_2^4\int_0^1 x^2y\, dxdy$.	(2)
	b)	Find the area enclosed by the lines $x = 0,\ y = 0,$ and $x + y = 1$	(3)
5	a)	Find the value of 'a' so that the vector $\vec{F} = (x + 3y)\hat{\imath} + (y - 2z)\hat{\jmath} + (x + az)\hat{k}$ is solenoidal.	(2)
	b)	Evaluate $\int_C y\,dx + x\,dy$ where C is the path $y = x^2$ from (0, 0) to (1, 1).	(3)
6	a)	Determine whether the vector field $\vec{F} = yz\,\hat{\imath} + xz\,\hat{\jmath} + xy\,\hat{k}$ is free of sources and sinks.	(2)
	b)	Use divergence theorem to find the outward flux of the vector field $\vec{F} = z^2\hat{\imath} - x^3\hat{\jmath} + y^3\hat{k}$ across the surface of the sphere $x^2 + y^2 + z^2 = 1$	(3)

PART B

Module I

Answer any two questions. Each question carries 5 Marks

7		Test the convergence of (i) $\sum_{k=1}^{\infty}\frac{(k+1)!}{2!\ k!\ 2^k}$ (ii) $\sum_{k=1}^{\infty}(\frac{k+1}{k})^{k^2}$	(5)

8 Find the Taylor series expansion of $f(x) = \sin \pi x$ about $x = \frac{1}{2}$. (5)

9 Find the radius of convergence and interval of convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}x^k$ (5)

Module II

Answer any two questions. Each question carries 5 Marks

10 Find $\frac{dz}{dt}$ using chain rule, if $z = 3x^2y^3$ where $x = t^4,\ y = t^3$. (5)

11 Find the local linear approximation of $f(x, y, z) = xyz$ at the point $(1, 2, 3)$. (5)

12 Locate all relative extrema and saddle points of $f(x, y) = 2xy - x^3 - y^2$. (5)

Module III

Answer any two questions. Each question carries 5 Marks

13 If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \|\vec{r}\|$ then prove that $\nabla r^n = nr^{n-2}\vec{r}$. (5)

14 Find the equation of the tangent plane and parametric equations of the normal line to the surface $x^2 + y^2 + 4z^2 = 12$ at the point $(2, 2, 1)$. (5)

15 Suppose that a particle moves through 2-space so that its position vector $\vec{r}(t) = (t^2 - 2t)\,\hat{\imath} + (t^2 - 4)\,\hat{\jmath}$. Find the scalar tangential component and scalar normal component of acceleration at time $t = 1$. (5)

Module IV

Answer any two questions. Each question carries 5 Marks

16 Using polar co-ordinates, evaluate $\iint_R e^{-(x^2+y^2)}\, dA$ where R is the circle $x^2 + y^2 = 1$ (5)

17 By reversing the order of integration, evaluate $\int_0^{\pi}\int_x^{\pi}\frac{\cos y}{y}\,dy\,dx$ (5)

18 Find the volume of the solid within the cylinder $x^2 + y^2 = 4$ and between the planes $z = 0$ and $x + z = 1$. (5)

Module V

Answer any three questions. Each question carries 5 Marks

19 Find the work done by the force field $\vec{F} = xy\,\hat{\imath} + yz\,\hat{\jmath} + xz\,\hat{k}$ on a particle that moves along the curve $x = t,\ y = t^2,\ z = t^3$ from $t = 0$ to $t = 1$ (5)

20 Determine whether $\vec{F} = e^x\cos y\,\hat{\imath} - e^x\sin y\,\hat{\jmath}$ is conservative. If so, find a potential function for it. (5)

21 Evaluate $\int_C x^2\,dx + xy\,dy$ along the curve given by $x = 2\cos t, y = 2\sin t$, $0 \le t \le \pi$ (5)

22 Find $\nabla.(\nabla \times \vec{F})$ and $\nabla \times (\nabla \times \vec{F})$ if $\vec{F} = x\,\hat{\imath} + xy\,\hat{\jmath} + xyz\,\hat{k}$ (5)

23 Show that $\int_{(0,0)}^{(3,2)} 2xe^y\,dx + x^2e^y\,dy$ is independent of the path. Also find the value of the integral. (5)

Module VI

Answer any three questions. Each question carries 5 Marks

24 Use Green's theorem to evaluate $\int_C 2xy\,dx + (x^2 + x)dy$ where C is the triangle with vertices (0, 0) (1, 0) and (1, 1) (5)

25 Apply Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = x^2\hat{\imath} + 4xy^3\hat{\jmath} + y^2x\,\hat{k}$ where C is the rectangle $0 \le x \le 1,\ 0 \le y \le 3$ in the plane $z = y$. (5)

26 Evaluate the surface integral $\iint_\sigma\ ds$ where σ is the part of the plane $x + y + z = 1$ that lies in the first octant. (5)

27 Apply Green's theorem to evaluate $\int_C xcosy\,dx - ysinx\,dy$ where C is the boundary of the square formed by $x = 0,\ x = \pi,\ y = 0,\ y = \pi$. (5)

28 Use divergence theorem to find the outward flux of the vector field $\vec{F} = xy\,\hat{\imath} + yz\,\hat{\jmath} + xz\,\hat{k}$ across the surface of the cube bounded by the planes $x = 0, x = 2,\ y = 0,\ y = 2,\ z = 0,\ z = 2.$ (5)

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