C 6150 (Pages : 2) Name..................................

Reg. No..................................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2010

ME 04 605—OPERATIONS RESEARCH

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Show that the vectors (2, – 2, 1), (1, 4, – 1) and (4, 6, – 3) are linearly independent.

(b) Find the rank of the matrix

$$\begin{pmatrix} 2 & -1 & 3 & 0 & 3 \\ 2 & 6 & -4 & -14 & 10 \\ 2 & -1 & 3 & 0 & 3 \\ 0 & 1 & -1 & -2 & 1 \end{pmatrix}$$

(c) Briefly explain the various steps in Charne's M-method.

(d) Explain dual theory of linear programming with an example.

(e) Explain the main steps of UV-method in solving a transportation problem.

(f) Explain the relationship between an assignment problem with a transportation problem.

(g) Describe the different types of service time distribution in queues.

(h) Explain briefly M/M/1 queue.

($8 \times 5 = 40$ marks)

II. (a) In what value of α, the vectors (1, 2, 9, 8), (2, 0, 0, α), (α, 0, 2, 8) and (0, 1, 1, 0) are (i) linearly dependent ; (ii) linearly independent.

Or

(b) Find the value of λ and μ for which the system of equations $x + 2y + z = 3$, $2x + 3y - z = 4$, $5x + \lambda y - z = \mu$ has (i) unique solution ; (ii) no solution ; (iii) many solutions.

III. (a) Solve the following LP problem by Simplex method :

$$\text{Maximize } Z = 10x_1 + 8x_2 + 15x_3$$

$$\begin{aligned} \text{subject to } \quad 2x_1 + x_2 + 3x_3 &\leq 10 \\ x_1 + 4x_2 + x_3 &\leq 16 \\ x_1 - x_3 &\leq 2 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

Or

Turn over

(b) Solve the following LP problem :—

Minimize $Z = 5x_1 + 6x_2$

$$x_1 + 2x_2 \geq 15$$
$$3x_1 + x_2 \geq 10$$
$$x_1 + x_2 \geq 9$$
$$x_1, x_2 \geq 0.$$

IV. (a) Solve the following transportation problem by Vogel's Approximation Method :

Destination

		1	2	3	4	5	
Source	1	12	11	15	17	21	50
	2	8	7	5	4	6	100
	3	17	19	18	21	25	250
		70	140	50	100	40	

Or

(b) Write down the primal and dual LP problems for the following 3 × 3 game. Obtain the optimal strategies :

Player B

$$\text{Player A} \begin{pmatrix} 0 & 2 & 2 \\ 3 & -1 & 3 \\ 4 & 4 & -2 \end{pmatrix}$$

V. (a) Customers arrive at a bank with a single teller at a rate of 15 per hour randomly. The service times are exponentially distributed with a mean of 3 minutes. Assume the Poisson process for the arriving customers, find (i) the average number of customers in the bank ; (ii) the mean waiting time of a customer at the counter ; (iii) fraction time the numbers customers in the bank is less 5.

Or

(b) Solve the following dynamic programming problem :

Maximize $Z = x_1^2 + 2x_2^2 + 3x_3^2$

$$x_1 + x_2 + x_3 = 50$$
$$x_1, x_2, x_3 \geq 0.$$

(4 × 15 = 60 marks)