



Course Code: CST306

Course Name: ALGORITHM ANALYSIS AND DESIGN

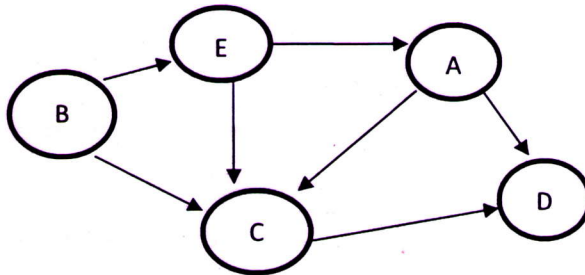
Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

- | | | |
|---|---|-----|
| 1 | Explain the best case and worst case of a search operation in a binary search tree. | (3) |
| 2 | Is $n^2 \in o(n^2)$? (Little o). Justify your answer. | (3) |
| 3 | Explain about any two cases of rotations in an AVL-Tree with proper example. | (3) |
| 4 | Obtain the topological sort order of the given graph $G=(V,E)$ | (3) |



- | | | |
|----|---|-----|
| 5 | Give the control abstraction of divide and conquer algorithm design strategy. | (3) |
| 6 | Consider an instance of fractional knapsack problem with 3 objects and capacity 20Kg. The profit and weight vectors corresponding to the 3 objects respectively are (10, 15, 20) and (8, 10, 20). Compute an optimal solution to this instance of fractional knapsack problem. What will be the profit earned corresponding to this solution? | (3) |
| 7 | What are the main steps involved in solving a computational problem using the algorithm design strategy Dynamic Programming. | (3) |
| 8 | List any three differences between Backtracking and Branch and Bound strategies. | (3) |
| 9 | Let $G = (V, E)$ be a graph with vertex set V and edge set E . Define the optimization and decision versions of CLIQUE problem in G . | (3) |
| 10 | Compare Las Vegas Algorithm and Monte Carlo algorithms. | (3) |

PART B

Answer one full question from each module, each carries 14 marks.

Module I

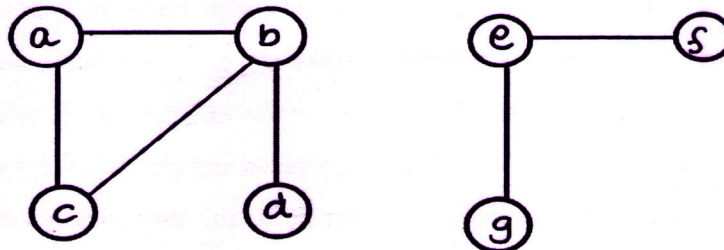
- 11 a) Illustrate the best case, worst case and average case of linear search algorithm (7)
with proper examples.
- b) Solve the following recurrence using recursion tree method. (7)
 $T(n) = 2T(n/2) + 1$

OR

- 12 a) Solve the following recurrence using substitution method. (8)
 $T(n) = 3T\left(\frac{n}{4}\right) + cn^2$
- b) Solve the following recurrences using Master's theorem. (6)
- i) $T(n) = 2T\left(\frac{n}{4}\right) + 1$
- ii) $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

Module II

- 13 a) Create an AVL tree by inserting the values from 1 to 8 in increasing order. (7)
- b) Give an algorithm for computing connected components in a graph using (7)
Disjoint Set data structure operations. Explain the working of this algorithm on
the graph given below.

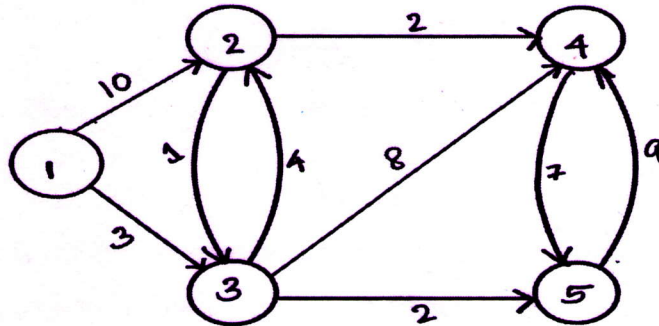


OR

- 14 a) Give an appropriate algorithm for Depth First Search in a given connected (9)
graph. Derive the time complexity of the algorithm with proper justification.
- b) What are strongly connected components of a directed graph? Explain with an (5)
example.

Module III

- 15 a) Apply Dijkstra's algorithm on the given graph to find the shortest path to all destinations starting from vertex with label 1 (6)



- b) Write Kruskal's algorithm to find the minimum cost spanning tree of a given weighted connected graph. What is the time complexity of the algorithm? Justify your answer. (8)

OR

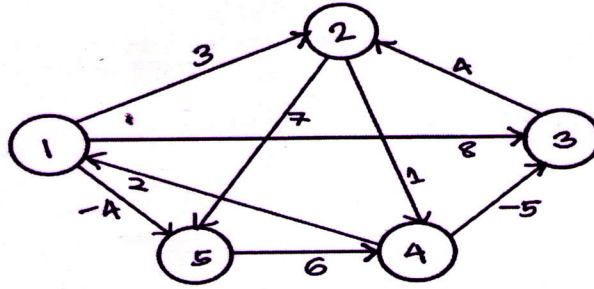
- 16 a) Formally state the fractional knapsack problem. Give an algorithm for computing an optimal solution to this problem using greedy strategy. What is its time complexity? Justify your answer. (9)
- b) Give the control abstraction of greedy algorithm design strategy. (5)

Module IV

- 17 a) Given a chain of 4 matrices $\langle A_1, A_2, A_3, A_4 \rangle$ with dimensions $\langle 5 \times 4 \rangle, \langle 4 \times 6 \rangle, \langle 6 \times 2 \rangle,$ and $\langle 2 \times 7 \rangle$. Fully parenthesize the given matrices so that the number of scalar multiplications required to compute their product is minimum. What is the value of optimum number of scalar multiplicationsto compute the product of the given chain of matrices? (9)
- b) Define the control abstraction for Backtracking (5)

OR

- 18 a) Consider the weighted directed graph given in the following Figure. (8)



Construct the weight adjacency matrix of the given graph. Apply the Floyd-Warshall algorithm to construct the matrix D^2 that represents the shortest paths distance between all vertices i and j ($1 \leq i \leq 5$ and $1 \leq j \leq 5$) through intermediate vertices 1 and 2.

- b) Define n-queens problem. Draw the state space diagram of 4-queens problem by applying backtrack method. (6)

Module V

- 19 a) Let $G = (V, E)$ be a graph with vertex set V and edge set E . Prove that the decision version of Clique problem in G belongs to the complexity class NP. (5)
- b) Give the randomized version of Quick Sort and prove that the expected number of comparisons performed by this algorithm is $O(n \log_2 n)$ (9)

OR

- 20 a) Let $G = (V, E)$ be a graph with vertex set V and edge set E . What is meant by a vertex cover of G ? Also prove that the decision version of vertex cover problem is NP-Hard. (8)
- b) Define Bin Packing problem. Give the First Fit heuristic approximation for this problem. What is the approximation ratio of this heuristic? (6)
