6051 **Name:** ....

Reg.No. ....

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2010

EN.04.401 (a) – Engineering Mathematics – IV

(Common for all except CS and IT)
(2004 Admissions)

Time: Three hours **Maximum: 100 marks**

Answer **all** *questions.*

1. (a) Is the function $V = e^x (x \sin y + y \cos y)$ harmonic ?

   (b) Find the image of the region $1 < x < 2,\ 1 < y < 2$ under $w = z^2$.

   (c) Evaluate $\int_C \frac{dz}{\left(z^2 + 4\right)^2}$ where C is $|z - i| = 2$.

   (d) Find the Laurent's expansion of $\frac{z-1}{z-2}$ for $|z - 1| < 1$.

   (e) Show that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$.

   (f) Find the value of $8P_4(x) + 20P_2(x) + 7P_0(x)$.

   (g) Classify the equation $U_{xx} + xu_{yy} = 0,\ x \neq 0$ for all x, y. Also find its characteristic equation.

   (h) Derive one dimensional wave equation.

(8 × 5 = 40 marks)

II. (a) (i) Determine the analytic function whose imaginary part is $e^{-x}(x \sin y - y \cos y)$.

   (ii) Show that an analytic function with constant modulus is constant.

Or

   (b) Show that the transformation $w = \frac{i(1-z)}{1+z}$ maps the circle $|z| = 1$ into the real axis of the w-plane and the interior of the circle $|z| < 1$ into the upper half of the w-plane.

Turn over

III. (a) (i) Evaluate $\int_C \frac{z \sec z}{(1-z)^2}\, dz$, C is $|z| = 3$.

(ii) Find the resedue at $z = 0$ of $\frac{1+e^z}{z\cos z + \sin z}$.

Or

(b) Show that $\int_0^{2\pi} \frac{d\theta}{1 - 2p\sin\theta + p^2} = \frac{2\pi}{1-p^2} (0 < p < 1)$.

IV. (a) Show that $e^{\frac{1}{2}\left(t - \frac{1}{t}\right)x} = \sum_{n=-\infty}^{\infty} t^n \, J_n(x)$.

Or

(b) (i) Express the function $x^3 + 3x^2 - 5x + 2$ in terms of Legendre's polynomials.

(ii) Show that $(1 - x^2)\, P_n^1(x) = n\left[P_{n-1}(x) - x P_n(x)\right]$.

V. (a) A string of length l is initially at rest in equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x,t)$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to :

(i) u is not infinite for $t \to \infty$.

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0, x = l$.

(iii) $u = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$.

(4 × 15 = 60 marks)