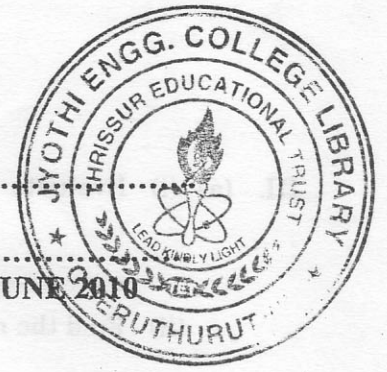


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Name:

Reg.No.

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, JUNE 2010
EN.04.401 (a) – Engineering Mathematics – IV



(Common for all except CS and IT)
(2004 Admissions)

Time: Three hours

Maximum: 100 marks

Answer all questions.

1. (a) Is the function $V = e^x (x \sin y + y \cos y)$ harmonic ?
- (b) Find the image of the region $1 < x < 2, 1 < y < 2$ under $w = z^2$.
- (c) Evaluate $\int_C \frac{dz}{(z^2 + 4)^2}$ where C is $|z - i| = 2$.
- (d) Find the Laurent's expansion of $\frac{z-1}{z-2}$ for $|z-1| < 1$.
- (e) Show that $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$.
- (f) Find the value of $8 P_4(x) + 20 P_2(x) + 7 P_0(x)$.
- (g) Classify the equation $U_{xx} + xu_{yy} = 0, x \neq 0$ for all x, y . Also find its characteristic equation.
- (h) Derive one dimensional wave equation.

(8 × 5 = 40 marks)

- II. (a) (i) Determine the analytic function whose imaginary part is $e^{-x} (x \sin y - y \cos y)$.
- (ii) Show that an analytic function with constant modulus is constant.

Or

- (b) Show that the transformation $w = \frac{i(1-z)}{1+z}$ maps the circle $|z| = 1$ into the real axis of the w -plane and the interior of the circle $|z| < 1$ into the upper half of the w -plane.

Turn over

III. (a) (i) Evaluate $\int_C \frac{z \sec z}{(1-z)^2} dz$, C is $|z| = 3$.

(ii) Find the residue at $z = 0$ of $\frac{1+e^z}{z \cos z + \sin z}$.

Or

(b) Show that $\int_0^{2\pi} \frac{d\theta}{1-2p \sin \theta + p^2} = \frac{2\pi}{1-p^2}$ ($0 < p < 1$).

IV. (a) Show that $e^{\frac{1}{2}(t-\frac{1}{t})x} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$.

Or

(b) (i) Express the function $x^3 + 3x^2 - 5x + 2$ in terms of Legendre's polynomials.

(ii) Show that $(1-x^2) P_n^1(x) = n [P_{n-1}(x) - x P_n(x)]$.

V. (a) A string of length l is initially at rest in equilibrium position and each of its points is given the

velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$. Find the displacement $y(x, t)$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ subject to :

(i) u is not infinite for $t \rightarrow \infty$.

(ii) $\frac{\partial u}{\partial x} = 0$ for $x = 0, x = l$.

(iii) $u = lx - x^2$ for $t = 0$ between $x = 0$ and $x = l$.

(4 × 15 = 60 marks)