Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S, FE) / S2 (PT) (S) / S4 (PT) (S) Examination January 2024 (2019 Scheme)

Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1 The probability distribution of a random variable is given by 3

$$f(x) = \frac{c}{2x};\ x = 1,2,3.$$

Find (i) the value of c (ii) $P[X \neq 1]$.

2 Derive the mean and variance of a Poisson distribution. 3

3 If a random variable X is uniformly distributed in the interval $[-c, c]$. Find the value of c such that $[X \geq 2] = 1/3$. 3

4 The joint density function of X and Y is given by 3

$$f(x,y) = \begin{cases} e^{-(x+y)}, & x \geq 0, y \geq 0 \\ 0, & otherwise \end{cases}$$

Check whether X and Y are independent or not.

5 The power spectral density of a wide sense stationary process is 3

$$S_X(\omega) = \begin{cases} 1, & -1 \leq \omega \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the autocorrelation function of the process.

6 If $X(t)$ is a WSS process with autocorrelation function given by $R_X(\tau) = \frac{16}{4\tau^2+4}$. Find the mean and variance of $X(t)$. 3

7 Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton – Raphson method. 3

8 Using Simpson's rule, compute $\int_0^{0.6} e^{-x^2} dx$ using six sub intervals. 3

9 Write the Normal equations to fit the parabola $y = a + bx + cx^2$. 3

10 Using Euler's method, find an appropriate value of y corresponding to $x = 0.04$, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$. Take $h = 0.02$. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Out of 600 families with 3 children each, how many families would be expected to have (i) 2 boys and 1 girl (ii) 3 boys (iii) 3 girls — 7

b) The joint density function of two random variables X and Y are given by — 7

$$f(x,y) = \begin{cases} c(2x+y), & x = 1,2\,; \quad y = 1,2 \\ 0, & otherwise \end{cases}$$

Compute (i) value of c (ii) Cumulative distribution functions of X and Y.

12 a) Show that Poisson distribution is a limiting case of Binomial distribution. — 7

b) The joint probability distribution of X and Y is given in the following table — 7

	Y=1	Y=2	Y=3	Y=4	Y=5	Y=6
X=0	0	0	1/32	2/32	2/32	3/32
X=1	1/16	1/16	1/8	1/8	1/8	1/8
X=2	1/32	1/32	1/64	1/64	0	2/64

Evaluate the following (i) $P[X \leq 1]$ (ii) $P[Y \leq 3]$ (iii) $P[X \leq 1, Y \leq 3]$ (iv) $P[X + Y \leq 4]$

Module -2

13 a) Let X follows and exponential distribution with pdf $f(x) = \begin{cases} \frac{1}{5}e^{-x/5}, x > 0 \\ 0, \quad otherwise \end{cases}$ — 7

Evaluate (i) $P[X > 5]$ (ii) $P[3 \leq X \leq 6]$ (iii) Mean (iv) Variance

b) The joint probability density function of two random variables X and Y is given — 7
by $f(x,y) = \begin{cases} \frac{1}{3}(3x^2 + xy), 0 < x < 1\,; 0 < y < 2 \\ 0, \quad otherwise \end{cases}$

Evaluate (i) $E[X]$ (ii) $E[Y]$ (iii) $P[X + Y \geq 1]$

14 a) The mean height of students follows a normal distribution with mean 68.22 inches and variance of 10.8 inches. How many students out 1000 would you expect to be over 72 inches height ? — 7

b) A distribution with unknown mean μ has variance equal to 1.5. Use Central Limit Theorem to determine how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. — 7

Module -3

15 a) A random process $X(t)$ is defined by $X(t) = 3\cos(4t + \theta)$, where θ is — 7

uniformly distributed in $[0, 2\pi]$. Find the mean, autocorrelation and auto covariance.

b) Find the power spectral density function of the WSS process whose autocorrelation function is given by $R_X(\tau) = \frac{a^2}{2}\cos(\omega\tau)$ 7

16 a) A random process $X(t)$ is defined by $X(t) = \sin(t + \theta)$, where θ is a random variable taking values $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ with equal probabilities. Check whether $X(t)$ is a WSS process or not. 7

b) Prove that sum of two independent Poisson processes is a Poisson process. 7

Module -4

17 a) Using Lagrange's interpolation method, find $y(10)$. Given that $y(5) = 12$, $y(6) = 13$, $y(9) = 14$, $y(11) = 16$. 7

b) Using Newton's divided difference interpolating formula find the missing value from the following data. 7

X	1	2	3	4	5	6
Y	14	15	-	5	6	19

18 a) Using Regula Falsi method, evaluate the fourth root of 32. 7

b) Compute the value of $\int_0^2 e^{x^2}\,dx$ using Trapezoidal Rule by taking 10 intervals. 7

Module -5

19 a) Solve the system of equations using Gauss-Seidel method. 7

$$4x + 2y + z = 14, \qquad x + 5y - z = 10, \qquad x + y + 8z = 20$$

Perform four iterations.

b) Use Runge-Kutta Method of fourth order to compute y for $x = 0.1$, given $\frac{dy}{dx} = \frac{xy}{1+x^2}$, $y(0) = 1$. Take $h = 0.1$ 7

20 a) Using method of least square fit a straight line of the form $y = ax + b$ for the following data 7

x	12	15	21	25
y	50	70	100	120

b) Using Adams-Moulton Method, solve $\frac{dy}{dx} = x^2(1 + y)$ for $x = 1.4$, Given that 7

$$y(1) = 1, \quad y(1.1) = 1.233, \quad y(1.2) = 1.548, \quad y(1.3) = 1.979$$

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