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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Second Semester B.Tech Degree (S, FE) Examination January 2024 (2019 Scheme)

Course Code: MAT 102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 SCHEME)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1 Find the parametric equation of the tangent line to the circular helix $x = \cos t, y = \sin t, z = t$, where $t = \pi$. (3)

2 If $\bar{f}(x, y, z) = x^2y\,\hat{\imath} + 2y^3z\,\hat{\jmath} + 3z\,\hat{k}$, find $div\,\bar{f}$. (3)

3 Use Green's theorem to evaluate $\int_c (x^2 - 3y)\,dx + 3x\,dy$, where C the circle $x^2 + y^2 = 4$. (3)

4 Show that $\bar{f} = (y + z)\,\hat{\imath} - xz^3\,\hat{\jmath} + x^2 \sin y\,\hat{k}$ is free of source and sink. (3)

5 Solve $y''' + 9y' = 0$. (3)

6 Find the Wronskian corresponding to the solution of $y'' - 2y' + y = 0$ (3)

7 Find the Laplace transform of $\sin^2 2t$ (3)

8 Find $L^{-1}\left\{\frac{1}{(s-1)(s-2)}\right\}$ (3)

9 Find the Fourier cosine integral representation of the function (3)

$$f(x) = \begin{cases} 1 & : if\ 0 < x < 1 \\ 0 & : \quad if\ x > 1 \end{cases}$$

10 Find the Fourier cosine transform of $e^{-x}, x > 0$ (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module I

11 a) Find the directional derivative of $f(x, y, z) = x^2y - yz^3 + z$ at the point $(1, -2, 0)$ in the direction of the vector $\bar{a} = 2\,\hat{\imath} + \hat{\jmath} - 2\hat{k}$. (7)

b) Evaluate $\int_c (3x^2 + y^2)\,dx + 2xy\,dy$, along the circular arc C given by $x = \cos t, y = \sin t, 0 \le t \le \frac{\pi}{2}$ (7)

OR

12 a) Find the work done by the force field $\bar{f} = xy\,\hat{\imath} + yz\,\hat{\jmath} + xz\,\hat{k}$ on a particle that moves along the curve $\bar{r}(t) = t\,\hat{\imath} + t^2 z\,\hat{\jmath} + t^3\,\hat{k}, 0 \le t \le 1$. (7)

b) Show that the force field $\bar{f} = y\,\hat{\imath} + x\,\hat{\jmath}$ is conservative. Hence evaluate $\int_{(0,0)}^{(1,1)} \bar{f}\,.d\bar{r}$ (7)

Module II

13 a) Evaluate the surface integral $\iint_\sigma xz\,ds$, where σ is the portion of the plane $x + y + z = 1$, that lies in the first octant. (7)

b) Use Stokes' theorem to evaluate $\int_c \bar{f}.d\bar{r}$, where $\bar{f} = (x - 2y)\hat{\imath} + (y - z)\hat{\jmath} + (z - x)\hat{k}$ and C is the circle $x^2 + y^2 = a^2$ in the xy-plane with counter clockwise orientation looking down the positive z axis. (7)

OR

14 a) Let σ be the portion of the surface $z = 1 - x^2 - y^2$ that lies above the xy-plane. Find the flux of the vector field $\bar{f} = x\,\hat{\imath} + y\hat{\jmath} + z\,\hat{k}$ across σ. (7)

b) Evaluate $\int_C tan^{-1}y\,dx - \frac{y^2x}{1+y^2}dy$, where C is the square with vertices $(0,0), (1,0), (0,1)$ and $(1,1)$. (7)

Module III

15 a) Solve the initial value problem $y'' + 9y = 0, y(0) = 0.2, y'(0) = -1.5$. (7)

b) By the method of variation of parameters to solve $y'' + 4y = \tan 2x$. (7)

OR

16 a) By the method of undetermined coefficients solve $y'' + 2y' + 4y = 3e^{-x}$. (7)

b) Solve $x^2y'' + xy' + 9y = 0, y(1) = 0, y'(1) = 2.5$. (7)

Module IV

17 a) Find the Laplace transform of (i) $t \sin 2t$ (ii) $e^{-t} \sin 3t\ \cos 2t$. (7)

b) Using convolution theorem find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$. (7)

OR

18 a) Find $L^{-1}\left\{\frac{4s+5}{(s+2)(s-1)^2}\right\}$. (7)

b) Use Laplace transforms to solve $y'' + 2y' + 2y = 0,\ y(0) = y'(0) = 1$. (7)

Module V

19 a) Find the Fourier transform of $f(x) = \begin{cases} 1: & \text{if } |x| < 1 \\ 0: & \text{otherwise} \end{cases}$. (7)

b) Find the Fourier sine integral of $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ (7)

OR

20 a) Using Fourier integral representation show that $\int_0^\infty \frac{\cos \omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x > 0.$ (7)

b) Find the Fourier sine transform of $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & x > a \end{cases}$ (7)

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