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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S6 (S, FE) / S4 (PT) (S, FE) Examination January 2024 (2015 Scheme)



Course Code: CS302

Course Name: DESIGN AND ANALYSIS OF ALGORITHMS

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

Marks

1 Analyse the complexity of the given function: (3)

```
void function(int n)
{
    int count = 0;
    for (int i=n/2; i<=n; i++)
        for (int j=1; j+n/2<=n; j = j++)
            for (int k=n; k>=1; k = k / 2)
                count++;
}
```

2 Find the worst-case complexity of the algorithm given below: (3)

```
Function(A) //A is an integer array
forj = 2 to length[A]
do key = A[j]
i = j - 1
while i > 0 and A[i] > key
do A[i + 1] = A[i]
i = i - 1
A[i + 1] = key
```

3  $n^k = O(c^n)$ ,  $k \geq 1$ ,  $c > 1$ . Comment on the given asymptotic relationship. Is it True or False? (3)

4 List the properties of Red Black Tree. (3)

**PART B**

*Answer any two full questions, each carries 9 marks.*

5 a) Write a pseudo-code for a divide-and-conquer algorithm for finding the position of the largest element in an array of  $n$  numbers. Set up and solve a recurrence relation for the algorithm. (4)

b) Solve the given recurrence using Iteration method: (5)

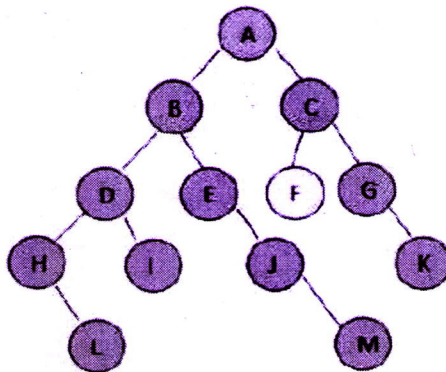
$$T(n) = 8T(n/2) + n^2$$

- 6 a) Solve the given recurrence using **Recursion Tree** method: (3)  
 $T(n) = T(n/4) + T(n/2) + n^2$
- b) State Master Theorem. (3)
- c) Can you solve the recurrence using **Master Theorem**? Justify. (3)  
 $T(n) = 2T(n/2) + n \log n$
- 7 a) Create a 2-3-4 tree by inserting the following values: (5)  
 R, Y, F, X, A, M, C, D, E, T, H, V, L, W, G (in that order). Show the tree after each 'node split'.
- b) Derive the best case and average case complexity of linear search. (4)

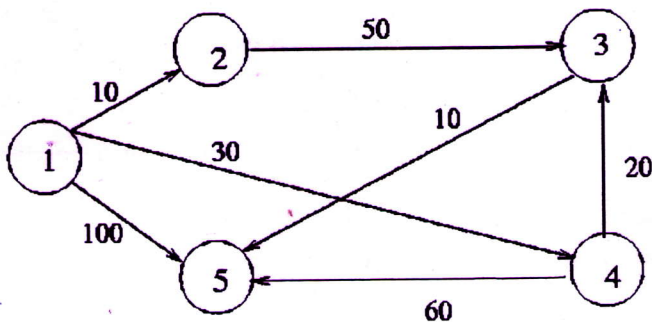
**PART C**

*Answer all questions, each carries 3 marks.*

- 8 Starting from the node 'A', which algorithm (DFS or BFS) will visit the least number of nodes before visiting the node 'F'? Justify. Assume that if there is ever a choice amongst multiple nodes, both the BFS and DFS algorithms will choose the left-most node first. (3)



- 9 Find shortest path distance to each node from Source node '1' using Dijkstra's algorithm. (3)

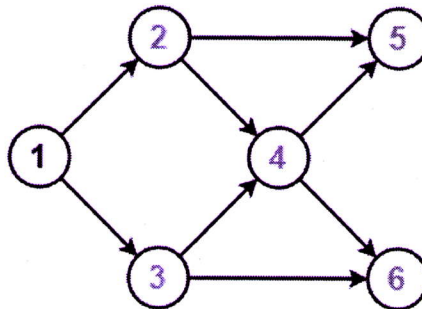


- 10 Give the control abstraction of Divide and Conquer algorithm. (3)
- 11 Set up and solve the recurrence relation for the number of key comparisons made by Mergesort in the worst case. (3)

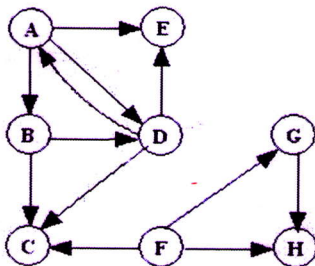
**PART D**

*Answer any two full questions, each carries 9 marks.*

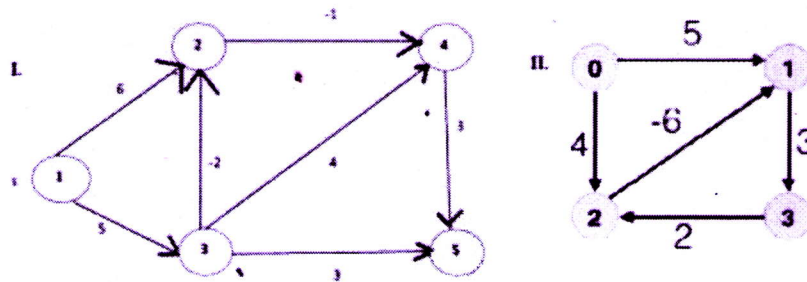
- 12 a) Provide any two possible scheduling of the given six jobs. Dependencies among jobs are represented by the following graph. (5)



- b) Let  $A_1, A_2, A_3$  and  $A_4$  be four matrices of dimensions  $5 \times 4, 4 \times 6, 6 \times 2,$  and  $2 \times 7,$  respectively. Find the minimum number of scalar multiplications required to get the product  $A_1 \times A_2 \times A_3 \times A_4$  using the basic matrix chain multiplication method (dynamic programming). (4)
- 13 a) Classify the given graph edges into Tree, Forward, Back and Cross edges. Resolve ties by considering alphabetical order. (4)



- b) Compare normal matrix multiplication, divide-and-conquer matrix multiplication and Strassen's matrix multiplication with respect to their time complexities. (3)
- c) "Dynamic Programming is all about Remembering the past". Justify the statement with Fibonacci number generation as example. (2)
- 14 a) Write Bellman-Ford algorithm. (2)
- b) Apply Bellman-Ford algorithm on the two graphs given below: For first graph assume node '1' as source and '0' for the second graph. (5)



- c) Comment on your results obtained from two graphs. (2)

**PART E**

*Answer any four full questions, each carries 10 marks.*

- 15 a) Compare Divide and Conquer with Dynamic Programming. (5)  
 b) Let  $G$  be an undirected connected graph. Consider the following statement (3)  
 S1: if  $G$  has a unique minimum spanning tree, then all the edge weights in the graph are distinct.

Whether the statement S1 is correct? Justify your answer

- c) Suppose we have a graph with negative weight edges. We take the largest magnitude negative edge weight  $-k$  and reset each edge weight  $w$  to  $w+k+1$ . Does the minimum spanning tree remain same in the modified graph? (2)
- 16 a) There are 5 cities in a network. The cost of building a road directly between  $i$  and  $j$  is the entry  $c(i,j)$  in the matrix below. An infinite entry indicates there is a mountain in the way and the road cannot be built. Find the least cost of making all the reachable cities. (5)

0	3	5	11	9
3	0	3	9	8
5	3	0	$\infty$	10
11	9	$\infty$	0	7
9	8	10	7	0

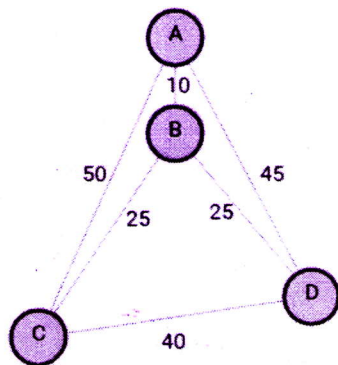
- b) Write the algorithm used for solving the above problem and analyse the time complexity of it. (5)
- 17 a) Write the control abstraction of Greedy algorithm. (2)  
 b) Write the Greedy algorithm for solving fractional knapsack problem and apply the same to the following instance where  $n=7$  and capacity  $(m)=15$ . (6)

Object	1	2	3	4	5	6	7
Profit	10	5	15	7	6	18	3
Weight	2	3	5	7	1	4	1

- c) Provide an example for a problem which cannot be optimally solved using Greedy strategy. (2)
- 18 a) Find the optimal solution for the following 0/1 knapsack problem instance using Backtracking.  $N=4$ , capacity ( $m$ ) =16 (5)

Object	1	2	3	4
Profit	40	30	50	10
Weight	2	5	10	5

- b) Show the state space tree for 4-queens problem using Backtracking for one possible solution. (5)
- 19 Explain the branch and bound strategy used to solve Travelling Salesman Problem and trace the Least Cost Branch and Bound algorithm for finding Travelling salesman tour of the graph given below. (10)



- 20 a) Define tractable and intractable problems with examples. (2)
- b) Discuss about four complexity classes each with proper examples. (5)
- c) How polynomial time reduction can be used to prove NP-Completeness. (3)

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