A 0800MAT201122101 Pages: 3

Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech S3 (R,S) / S1 (PT) (S, FE) Examination December 2023 (2019 scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Derive partial differential equation from $z = f(x+2t) + g(x-2t)$ (3)

2 Solve $xp + yq = z$. (3)

3 Find the steady state temperature distribution in a rod of length 30 cm if the ends are kept at $10^0\, C$ and $100^0\, C$. (3)

4 A tightly stretched string of length 50 cm has its ends fastened at $x = 0$ and $x = 50$. The midpoint of the string is then taken to height h and then released from rest in that position. Find the initial position of the string. (3)

5 Check whether the function $f(z) = \begin{cases} \frac{Re(z)}{1-|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at $z = 0$. (3)

6 Is $f(z) = \bar{z}$ analytic. Justify your answer. (3)

7 Evaluate $\int_0^{1+i} (x - iy)dz$ along the straight line path from 0 to 1+i (3)

8 Evaluate $\int_C \frac{dz}{(z^2+1)(z-1)}$ when C is $|z| = 0.5$ (3)

9 Find the residue of $z\cos\left(\frac{1}{z}\right)$ at $z = 0$ (3)

10 State Cauchy's integral formula and Cauchy's residue theorem (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11 (a) Form the partial differential equation by eliminating the arbitrary function f from the relation $xyz = f(x+y+z)$. (7)

(b) Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u$ with the initial condition $u(x,0) = e^{2x}$ by applying method of separation of variables. (7)

12 (a) Solve $\frac{\partial^2 u}{\partial x \partial y} = \cos(x)\cos(y)$ given that $\frac{\partial u}{\partial y} = e^{-y}$ when $x = 0$ and $u = 0$ when $y = 0$. (7)

(b) Solve $(y^2+z^2)p - xyq + xz = 0$ (7)

Module 2

13 (a) Derive one-dimensional wave equation. (7)

(b) Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = 0°C, u(1,t) = 0°C$ and the temperature initially is $u(x,0) = x(1-x)$. (7)

14 (a) A homogeneous rod of conducting material of length l has its ends kept at zero temperature and the temperature initially is $u(x,0) = f(x)$. Find the temperature $u(x,t)$ at any time. (7)

(b) Solve the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to $u(0,t) = 0, u(1,t) = 0, u(x,0) = 3\sin(2\pi x), \frac{\partial u}{\partial t}(x,0) = 0$. (7)

Module 3

15 (a) Find all points where the Cauchy-Riemann equations are satisfied for the function $f(z) = xy$ and where it is analytic. Give reasoning for your answers. (7)

(b) Show that $u = 2 + 3x - y + x^2 - y^2 - 4xy$ is harmonic. Also find its conjugate harmonic function v. (7)

16 (a) Show that the function $f(z) = \frac{1}{z}$ is analytic except at $z = 0$. (7)

(b) Find the image of the triangular region bounded by $x = 1, y = 1$ and $x + y = 1$ under the transformation $w = z^2$. (7)

Module 4

17 (a) Evaluate $\int_C \frac{1}{(z+4i)(z+i)} dz$, where C is $|z| = 2$ using Cauchy's integral formula. (7)

(b) Find the Maclaurin series expansion of $e^z \sin z$. (7)

18 (a) Evaluate $\int_C \frac{z^2+z+1}{(z-1)^2} dz$, where C is the circle $|z| = 3$ using Cauchy's integral formula. (7)

(b) Find the Taylor series expansion of $f(z) = \frac{1}{z^2}$ about $z = 2$. (7)

Module 5

19 (a) Obtain the Laurent series expansion of $\frac{1}{(z-2)(z-3)}$ in the region (7)

$(i)|z| < 1$ $(ii)\ 2 < |z| < 3$ $(iii)\ |z| > 3$

(b) Evaluate $\int_0^{\infty} \frac{1}{1+x^2} dx$. (7)

20 (a) Find the poles and residues of the function $\frac{1}{z^4-1}$ (7)

(b) Evaluate $\int_0^{2\pi} \frac{1}{3+2\sin\theta} d\theta$ (7)