A

0800MAT203122102

Pages: 3

Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (R,S) / S1 (PT) (S, FE) Examination December 2023 (2019 scheme)

Course Code: MAT203

Course Name: Discrete Mathematical Structures

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1 Using truth table, verify the logical equivalence (3)
$[p \rightarrow (q \vee r)] \Leftrightarrow [\neg r \rightarrow (p \rightarrow q)]$

2 Write the symbolic form of the following statement and its negation. Also express the negation in words. (3)
"If cow is black then milk is white."

3 State Pigeon hole principle. Let S be a set with positive integers and $|S| = 37$. How many elements of S have the same remainder upon division by 36? (3)

4 In how many ways can the letters of the word 'SOCIOLOGICAL' be arranged such that all the vowels are together? (3)

5 Let T be the set of all triangles in $\boldsymbol{R}^2$. Determine whether the relation R on T defined by, $t_1 \; R \; t_2$ if t_1 and t_2 have an angle of same measure, is reflexive, symmetric and transitive. (3)

6 Define a poset with an example. (3)

7 Differentiate between generating function and exponential generating function of a sequence. Write the generating function and exponential generating function of the sequence 1,1, 1...... (3)

8 A bank pays 6% interest on savings, compounding the interest yearly. Write the reccurence relation for this and solve it to find how much will a deposit of Rs 1000 be worth after 12 years. (3)

9 Is the set of integers Z, a semigroup under subtraction? Justify your answer. (3)

10 Show that the inverse of an element in a group is unique. (3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11(a) Check the validity of the argument. "If horses or cows eat grass, then mosquito is the national bird. If mosquito is the national bird then peanut butter tastes good on hot-dogs. But peanut butter tastes terrible on hot-dogs. Therefore, cows didn't eat grass." (7)

(b) Write the symbolic form of the open statement 'If x is even, then x is not divisible by 5'. Also write its converse, inverse and contrapositive both in symbolic form and in words. (7)

12(a) Establish the validity of the argument (7)

$$p \to (q \to r)$$
$$p \vee s$$
$$t \to q$$
$$\neg s$$
$$\overline{\therefore\ \neg r \to \neg t}$$

(b) Write the dual of the statement $(\neg p \wedge \neg q) \vee (T_0 \wedge p) \vee p$ and use laws of logic to show that it is equivalent to $p \wedge \neg q$. (7)

Module 2

13(a) Determine the number of six-digit integers (no leading zeros) in which (8)

(i) no digit may be repeated

(ii) digits may be repeated

(iii) digits may be repeated and the six-digit integer is even

(iv) no digit may be repeated and the six digit integer is divisible by 5

(b) In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun, or byte occurs? (6)

14(a) A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if (8)

(i) there are no restrictions

(ii) there must be six men and six women

(iii) there must be even number of women

(iv) there must be more women than men

(b) Determine the mnumber of positive integers $n, 1 \leq n \leq 2000$ that are not divisible by 2,3 or 5. (6)

Module 3

15(a) If $A = \{1,2,3,4\}$, $B = \{2.5\}$, $C = \{3,4,7\}$, determine (6)

(i) $A \cup (B \times C)$,

(ii) $(A \cup B) \times C$

(iii) $(A \times C) \cup (B \times C)$

(b) For $A = \{1,2,3,4,5\}$, $A_1 = \{1,2\}$, $A_2 = \{1,2,3\}$, $A_3 = \{2,3\}$, $A_4 = \{2,3,4,5\}$ and $B = \{w, x, y, z\}$, let $f: A \rightarrow B$ be given by $f = \{(1, w), (2, x), (3, x), (4, y), (5, y)\}$. Find $f(A_1)$, $f(A_2)$, $f(A_3)$, $f(A_4)$. (8)

16(a) In a distributive lattice, show that $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ together imply that $b = c$. (7)

(b) Draw the Hasse diagram of D_{42}. Find the complements of each of its elements. (7)

Module 4

17(a) Solve the recurrence relation, (7)

$a_{n+2} + 3a_{n+1} + 2a_n = 3^n, n \geq 0, a_0 = 0, a_1 = 1$

(b) Find the coefficient of x^5 in $(1 - 2x)^{-7}$ (7)

18(a) Find the solution of the recurrence relation, (7)

$a_{n+2} + 4a_{n+1} + 4a_n = 3^n, n \geq 0, a_0 = 1, a_1 = 2$

(b) Determine the coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \cdots)^4$ (7)

Module 5

19(a) Define a group with an example (7)

(b) Show that a cyclic monoid is abelian. (7)

20(a) Prove that the set of idempotent elements of M for any abelian monoid $(M, *, e)$ forms a submonoid. (7)

(b) Show that any group G is abelian iff $(ab)^2 = a^2b^2$ for all $a, b \in G$. (7)

*****