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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

First Semester B.Tech Degree Regular and Supplementary Examination December 2023 (2019 scheme)

Course Code: MAT 101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks

Marks

1 Find the rank of the matrix $\begin{bmatrix} 3 & 5 & 0 \\ 0 & 0 & 5 \\ 5 & 0 & 0 \end{bmatrix}$ (3)

2 Show that the quadratic form $Q = 3x_1^2 + 22x_1x_2 + 3x_2^2$ is indefinite. (3)

3 Find $f_x(1,3)$ and $f_y(1,3)$ if $f(x,y) = 2x^3y^2 + 2y + 4x$. (3)

4 If $z = x^2y^2$ where $x = t^4, y = t^3$ find $\frac{dz}{dt}$ using chain rule. (3)

5 Evaluate $\int_0^1\int_0^2 (x+5)dydx$. (3)

6 Evaluate $\iint_R \frac{\sin x}{x} dA$, where R is the triangular region bounded by the x −axis, $y = x$ and $x = 1$. (3)

7 Determine whether the series $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k$ converges and if so find its sum (3)

8 Examine the convergence of $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ (3)

9 Find the Taylor series expansion of $f(x) = \frac{1}{x}$ about $x = -1$ (3)

10 Find the half range sine series representation of $f(x) = k$ in $(0, \pi)$ (3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following linear system of equations using Gauss elimination method (7)

$$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$

b Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (7)

12	a	Solve the following linear system of equations using Gauss elimination method. $$3x - 11y - 2z = -6$$ $$4y + 4z = 24$$ $$6x - 17y + z = 18$$	(7)
	b	Find the matrix of transformation that diagonalize the matrix $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$. Also write the diagonal matrix.	(7)

MODULE 2

13	a	If $w = x^2 + y^2 - z^2$ where $x = \rho sin\emptyset cos\theta, y = \rho sin\emptyset sin\theta, z = \rho cos\emptyset$. Find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ using chain rule.	(7)
	b	Locate all relative extrema and saddle points of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$	(7)
14	a	Show that the function $f(x,y) = 2\tan^{-1}(\frac{y}{x})$ satisfies the Laplace equation $f_{xx} + f_{yy} = 0$.	(7)
	b	Find the local linear approximation L of $f(x,y) = \ln(xy)$ at the point $P(1,2)$. Compute the error in approximation f by L at the point $Q(1.01, 2.01)$.	(7)

MODULE 3

15	a	Evaluate $\iint_R (3x - 2y)dA$, where R is the region enclosed by the circle $x^2 + y^2 = 1$.	(7)
	b	Evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy\, dx$ by reversing the order of integration.	(7)
16	a	Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y(x^2 + y^2) dxdy$ using polar coordinates.	(7)
	b	Let G be the tetrahedron in the first octant bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, (a, b, c > 0)$ find the volume of G.	(7)

MODULE 4

17	a	Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{4k^2-2k+6}{8k^7+k-8}$ (ii) $\sum_{k=1}^{\infty} (\frac{k+1}{k})^{k^2}$	(7)
	b	Test the convergence of the series $1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \frac{1.2.3.4}{1.3.5.7} + \cdots\cdots\cdots$	(7)
18	a	Show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ is conditionally convergent	(7)
	b	Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{(k+3)!}{3!\ k!\ 3^k}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$	(7)

MODULE 5

19	a	Find the Fourier series expansion of $f(x) = x + x^2$ in the range$(-\pi, \pi)$.	(7)

	b	Obtain the half range Fourier sine series of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4 - x, & 2 < x < 4 \end{cases}$	(7)
20	a	Find the Fourier series expansion of $f(x) = \lvert x \rvert$ in the range $(-\pi, \pi)$. Hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$	(7)
	b	Obtain the half range Fourier cosine series of $f(x) = x^2$ in $0 < x < 2$	(7)

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