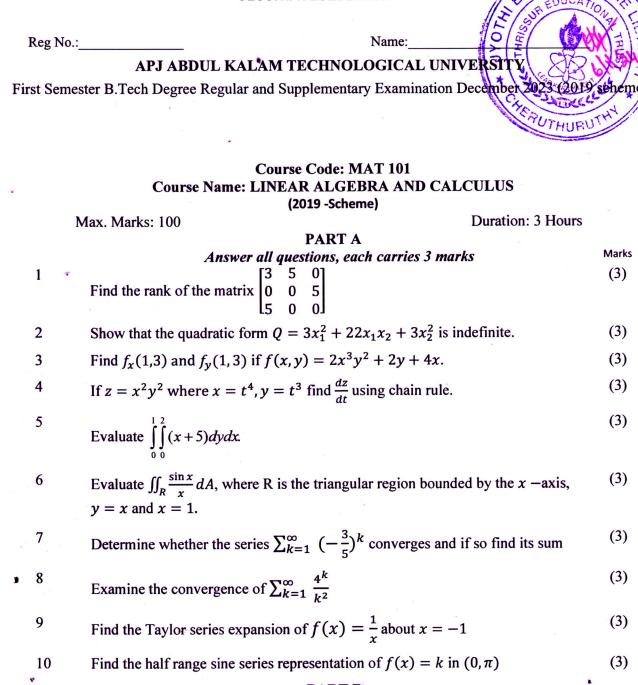
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PARTB

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 a Solve the following linear system of equations using Gauss elimination method (7) x + y - z = 9 8y + 6z = -6 -2x + 4y - 6z = 40

b Find the eigenvalues and eigenvectors of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (7)

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12 a Solve the following linear system of equations using Gauss elimination method. (7) 3x - 11y - 2z = -6

$$4y + 4z = 24$$
$$6x - 17y + z = 18$$

b Find the matrix of transformation that diagonalize the matrix $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$. Also write the diagonal matrix. (7)

MODULE 2

- 13 a If $w = x^2 + y^2 z^2$ where $x = \rho sin \phi cos \theta$, $y = \rho sin \phi sin \theta$, $z = \rho cos \phi$. Find (7) $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ using chain rule.
 - b Locate all relative extrema and saddle points of $f(x, y) = x^3 + 3xy^2 15x^2 15y^2 + 72x$ (7)
- 14 a Show that the function $f(x, y) = 2 \tan^{-1}(\frac{y}{x})$ satisfies the Laplace equation (7) $f_{xx} + f_{yy} = 0$.
 - b Find the local linear approximation L of $f(x, y) = \ln(xy)$ at the point P(1, 2). (7) Compute the error in approximation f by L at the point Q(1.01, 2.01).

MODULE 3

15 a Evaluate $\iint_R (3x - 2y) dA$, where R is the region enclosed by the circle (7)

$$x^2 + y^2 = 1.$$

(7)

- b Evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$ by reversing the order of integration.
- 16 a Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} y(x^2 + y^2) dx dy$ using polar coordinates. (7)
 - b Let G be the tetrahedron in the first octant bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, (a, b, c > 0) find the volume of G.

MODULE 4

- 17 a Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{4k^2 2k + 6}{8k^7 + k 8}$ (ii) $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2}$ (7)
 - b Test the convergence of the series $1 + \frac{1.2}{1.3} + \frac{1.2.3}{1.3.5} + \frac{1.2.3.4}{1.3.5.7} + \dots$ (7)
- 18 a Show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ is conditionally convergent (7)
 - b Test the convergence of (i) $\sum_{k=1}^{\infty} \frac{(k+3)!}{3! \ k! \ 3^k}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$ (7)

MODULE 5

19 a Find the Fourier series expansion of $f(x) = x + x^2$ in the range $(-\pi, \pi)$. (7)

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- b Obtain the half range Fourier sine series of $f(x) = \begin{cases} x, & 0 < x < 2 \\ 4 x, & 2 < x < 4 \end{cases}$ (7)
- 20 a Find the Fourier series expansion of f(x) = |x| in the range $(-\pi, \pi)$. Hence show that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$
 - b Obtain the half range Fourier cosine series of $f(x) = x^2$ in 0 < x < 2 (7)
