

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S5 (S, FE) / S5 (PT) (S, FE) Examination December 2023 (2015 Scheme)

**Course Code: CS309****Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 3 marks.*

Marks

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| 1 | Explain the terms incidence and degree with examples for each | (3) |
| 2 | Differentiate path and circuit | (3) |
| 3 | A non hamiltaneon graph can have a hamiltaneon path. Say yes or no with reason | (3) |
| 4 | What s Travelling Salesman Problem | (3) |

PART B*Answer any two full questions, each carries 9 marks.*

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| 5 | a) What is Konigsberg bride problem? How you identify it is not possible to find solution for the problem? | (5) |
| | b) What is seating problem? | (4) |
| 6 | a) Differentiate isolated vertex and pendent vertex | (4) |
| | b) State and prove Dirac's theorem | (5) |
| 7 | a) What is the necessary and sufficient condition for a graph to be Euler? And also prove it | (5) |
| | b) What are the different types of digraphs? | (4) |

PART C*Answer all questions, each carries 3 marks.*

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| 8 | Prove that in a graph G, if there is exactly one path between every pair of vertices, then G is a tree | (3) |
| 9 | Sketch two different binary trees on 13 vertices, one having maximum height and other having minimum height | (3) |
| 10 | Draw the two simplest non planar graphs and also mention their properties.- | (3) |
| 11 | Prove that every circuit has even number of edges in common with any cut set | (3) |

PART D*Answer any two full questions, each carries 9 marks.*

- 12 a) Define a tree. Give any 4 properties of trees (5)
 b) Define binary tree and sketch all binary trees with 6 pendent vertices (4)
- 13 a) Define spanning tree. Show that the edges forming a spanning tree in a planar graph G correspond to the edges forming a set of chords in the dual G^* (5)
 b) Define rank and nullity of a graph G with examples (4)
- 14 Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge-disjoint union of cut-sets. (9)

PART E*Answer any four full questions, each carries 10 marks.*

- 15 a) Define the adjacency matrix $X(G)$ of a graph. Let X_r be adjacency matrix of a simple graph G , then prove that ij th entry in X_r is the number of different edge sequences of r edges between vertices V_i and V_j (6)
 b) Prove the theorem: (4)
 If $A(G)$ is an incidence matrix of a connected graph G with n vertices, the rank of $A(G)$ is $n-1$
- 16 a) Let A and B be, respectively, the circuit matrix and incidence matrix of a self-loop-free graph G . Prove that $A \times B^T = 0 \pmod{2}$ (5)
 b) Define cut-set matrix and list down any four properties of cut-set matrix (5)
- 17 Write Dijkstra's Shortest path algorithm and apply this on an example to find the shortest path (10)
- 18 How can two linear arrays be used to represent a digraph. Give an example. (10)
 Compare this representation with edge list representation in terms of storage.
- 19 a) Define the incidence matrix of a graph G . Prove that the rank of an incidence matrix of a connected graph with n vertices is $n-1$. (6)
 b) If B is a circuit matrix of a connected graph G with e edges and n vertices, then show that the number of linearly independent rows in $B = e-n+1$ (4)
- 20 Draw the flowchart of Connectedness and Components algorithm and also apply this algorithm on any graph (G) with 2 components (10)
