Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S, FE) S2 (S, FE) Examination December 2023 (2015 Scheme)

Course Code: MA 101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 5 Marks Marks

1 a) Determine whether the series $\sum_{k=1}^{\infty}\frac{1}{3^{k+1}}$ converges. If so find its sum. (2)

b) Use alternating series test, determine whether the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}$ converge or not (3)

2 a) Find the slope of the surface $z=\sqrt{3x+2y}$ in the y-direction at the point (2, 5). (2)

b) If $f(x,y,z)=x^3y^5z^7+xy^2+y^3z$. Find (i) f_{xx} (ii) f_{yy} (iii) f_{zz} (3)

3 a) Find the velocity vector of the particle, given the acceleration vector $\vec{a}(t)=sint\,\hat{\imath}+cost\,\hat{\jmath}+e^t\hat{k}$. (2)

b) Find the directional derivative of $f(x,y,z)=x^2y-yz^3+z$ at the point (1, -2, 0) in the direction of the vector $\vec{a}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$ (3)

4 a) Evaluate $\int_0^1\int_0^1\int_0^1 xyz\,dxdydz$. (2)

b) Find the area enclosed by the parabolas $y^2=x$ and $x^2=y$. (3)

5 a) Find the divergence of the vector field $\vec{F}=x^2\hat{\imath}-3y\hat{\jmath}-z^3\hat{k}$ (2)

b) Evaluate $\int_C y^2dx+x^2dy$ where C is the path $y=x$ from (0, 0) to (1, 1). (3)

6 a) Determine whether the vector field $\vec{F}=yz\,\hat{\imath}-xz^3\hat{\jmath}+x^2siny\,\hat{k}$ is free of sources and sinks. (2)

b) Apply Stoke's theorem to evaluate $\int_C x^2dx+y^2dy+z^2dz$ where C is the curve $z=\sqrt{x^2+y^2}$ below the plane $z=1$. (3)

PART B

Module I

Answer any two questions. Each question carries 5 Marks

7 Find the Taylor series expansion of $f(x)=\frac{1}{x}$ about $x=2$ (5)

8 Test the convergence of (i) $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ (ii) $\sum_{k=1}^{\infty}(\frac{2k-1}{3k+2})^k$ (5)

9 Find the radius of convergence and interval of convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x-5)^k}{5^k}$ (5)

Module II

Answer any two questions. Each question carries 5 Marks

10 Find the local linear approximation of $f(x,y,z) = \log(x+yz)$ at the point (2, 1, -1). (5)

11 If $u = f(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$ using chain rule. (5)

12 Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$. (5)

Module III

Answer any two questions. Each question carries 5 Marks

13 If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = \|\vec{r}\|$ then prove that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$. (5)

14 Find the unit tangent vector T(t) and unit normal vector N(t) to the graph $\vec{r} = 5cost\,\hat{\imath} + 5sint\,\hat{\jmath}$ at the point $t = \frac{\pi}{2}$. (5)

15 Find the equation of the tangent plane and parametric equations of the normal line to the surface $x^2+y^2+z^2 = 25$ at the point (-3, 0, 4). (5)

Module IV

Answer any two questions. Each question carries 5 Marks

16 By reversing the order of integration, evaluate $\int_0^1\int_x^1 \frac{1}{y(1+y^2)}\,dy\,dx$ (5)

17 Evaluate $\iint_R (x+y)\,dA$ where R is the region in the first quadrant of the circle $x^2+y^2 = 1$. (5)

18 Find the volume of the solid in the first octant bounded by the co-ordinate planes and the plane $x+y+z = 1$ (5)

Module V

Answer any three questions. Each question carries 5 Marks

19 Find $\nabla.(\nabla\times\vec{F})$ and $\nabla\times(\nabla\times\vec{F})$ if $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ (5)

20 Find the work done by the force field $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ along the path $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. (5)

21 Evaluate $\int_C (x^2+y^2)dx + dy$ where C is the curve given by $x = cost, y = sint,\ 0 \le t \le \frac{\pi}{2}$ (5)

22 Determine whether the vector field $\vec{F} = 3y^2\hat{\imath} + 6xy\,\hat{\jmath}$ is conservative. If so find its potential function. (5)

23 Show that $\int_{(1,2)}^{(4,0)} 3y\,dx + 3x\,dy$ is independent of the path. Also find the value of the integral. (5)

Module VI

Answer any three questions. Each question carries 5 Marks

24 Use Green's theorem to evaluate $\int_C (x - 2y)dx + (3x - y)dy$ where C is the boundary of the unit square. (5)

25 Use divergence theorem to find the outward flux of the vector field $\vec{F} = (x^2 + y)\hat{\imath} + z^2\hat{\jmath} + (e^y - z)\hat{k}$ where S is the surface of the rectangular solid bounded by the co-ordinate planes and the plane $x = 3,\ y = 1,\ z = 2$. (5)

26 Apply Green's theorem to evaluate $\int_C (-x^2y)dx + (y^2x)dy$ where C is the boundary of the region in the first quadrant enclosed by the circle $x^2 + y^2 = 16$ (5)

27 Evaluate the surface integral $\iint_\sigma x\,ds$ where $\sigma$is the part of the plane $x + y + z = 2$ that lies in the first octant. (5)

28 Apply Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F} = xz\,\hat{\imath} + 4x^2y^2\hat{\jmath} + xy\,\hat{k}$ where C is the rectangle $0 \le x \le 1,\ 0 \le y \le 3$ in the plane $z = y$. (5)

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