Reg No.:_______________ Name:_______________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (R,S) / S4 (PT) (R,S) / S2 (PT) (R,S) Examination June 2023 (2019 Scheme)

Course Code: MAT 204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 The following table gives the probabilities that a certain computer will malfunction 0,1,2,3,4,5 or 6 times on any day 3

No.of Malfunctions (x)	0	1	2	3	4	5	6
Probability f(x)	0.17	0.29	0.27	0.16	0.07	0.03	0.01

Find the mean and variance of the probability distribution.

2 Let X and Y have the following probability distribution. 3

X\Y	1	2
1	0.15	0.10
2	0.30	0.20
3	0.15	0.10

Check whether X and Y are independent or not.

3 Derive the mean and variance of an Exponential distribution. 3

4 The time for a superglue to set can be treated as a random variable having a normal distribution with mean 30 seconds. Find its standard deviation if the probability is 0.20 that it will take on a value greater than 39.2 seconds. 3

5 State any three properties of autocorrelation function. 3

6 Find the power spectral density of the WSS process with autocorrelation function $R(\tau) = 3e^{-2|\tau|}$. 3

7 Using Newton Raphson Method find a positive root of $x^4 - x - 10 = 0$ correct to 3 decimal places. 3

8 Using Trapezoidal rule evaluate the integral $\int_1^2 \frac{dx}{1+x^2}$ with h = 0.5. 3

9 By method of least squares, Fit the straight line of the form $y = ax + b$ for 3

x	1	2	3	4	5
y	5	7	9	10	11

10 Using Second order Runge-Kutta Method, evaluate $y(0.1)$ with h=0.1 from $\frac{dy}{dx} = y - \frac{2x}{y}$, y(0) =1. 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) The probability mass function of a discrete random variable is f(x) = kx, x= 1,2,3 where k is a positive constant. Find (i) the value of k (ii) $P(X \leq 2)$ (iii) E(X) and (iv) Var (1-X). 7

b) If X follows a Poisson distribution such that $P(X = 1) = 3/10$ and $P(X = 2) = 1/5$, Find $P(X = 4)$ and $P(X = 0)$ 7

12 a) The probability of a bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If 6 bombs are aimed at the bridge, find the probability that the bridge is destroyed. 7

b) If the joint probability density function of random variables X and Y is given by $f(x, y) = c(2x + 3y)$, x = 0,1,2; y = 1,2,3. Find (i) the value of c (ii) the marginal probability distributions of X and Y (iii) the probability distribution of X+Y. 7

Module -2

13 a) The cumulative distribution function of a continuous variable X is given by 7

$$F(x) = \begin{cases} 0, x \leq 2 \\ c(x-2), 2 < x < 6 \\ 1, x \geq 6 \end{cases}$$

Find (i) p.d.f (ii) value of c (iii) $P(1 \leq X \leq 5)$

b) Buses arrive at a specified stop at 15 min. intervals starting at 7 a.m. If a passenger arrives at the stop at a random time that is uniformly distributed between 7:00 and 7:30 a.m., find the probability that he waits (i) less than 5 minutes for a bus (ii) at least 12 minutes for a bus. 7

14 a) Examine whether the random variables X and Y are independent, whose joint probability density function is $f(x, y) = xe^{-x(y+1)}, 0 < x; y < \infty$ 7

b) A distribution with unknown mean μ has variance equal to 1.5. By using central limit theorem, find how large a sample should be taken in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. 7

Module -3

15 a) $X(t)$ is a random process defined by $X(t) = A\cos(\omega t + \theta)$ where ω is a constant and A and θ are independent random variables uniformly distributed in (-k, k) and $(-\pi, \pi)$ respectively. (i) Find the mean, (ii) autocorrelation(iii) Is $X(t)$ WSS ? 7

b) If X(t) is a WSS process with autocorrelation $R_X(\tau) = \frac{\tau^2+44}{\tau^2+4}$. Find the mean and variance. 7

16 a) Assume that X(t) is a random process defined as follows: X(t) = A cos ($2\pi t + \emptyset$) where A is a zero-mean normal random variable with variance $\sigma_A^2 = 2$ and $\emptyset$ is uniformly distributed random variable over the interval $-\pi \leq \varphi \leq \pi$. A and φ are statistically independent. Let the random variable Y be defined as $Y = \int_0^1 X(t)dt$. Determine (i) the mean of Y (ii) the variance of Y. 7

b) If people arrive at a book stall in accordance with a Poisson process with a mean rate of 3 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 minute, (ii) between 1 minute and 2 minutes (iii) 4 minutes or less. 7

Module -4

17 a) Solve the equation $3x + \sin x - e^x = 0$ by Regula Falsi Method. 7

b) Using Lagrange's interpolation method find y (4), Given the following data. 7

x	1	3	5	7
y	24	120	336	720

18 a) Evaluate y (2) from the following data using Newton's forward difference method. 7

x	0	5	10	15
y	14	379	1444	3584

b) Find y (3) Using Newton's divided difference formula, Given $y(1) = -26, y(2) = 12, y(4) = 256, y(6) = 844$. 7

Module -5

19 a) Solve the equations 4x + 2y + z = 14, x + 5y - z = 10, x + y +8z = 20, using Gauss-Siedel Method. 7

b) Using Euler's method, solve $\frac{dy}{dx} = x + y + xy$, given $y(0) = 1$, evaluate y at $x = 0.1$ by taking $h = 0.05$. 7

20 a) Apply fourth order Runge Kutta Method to find y (0.1) for the initial value problem $\frac{dy}{dx} = x + y^2, y(0) = 1$. 7

b) Solve $\frac{dy}{dx} = x^2(1 + y)$ for x =1.4 using Adam's Moulton Method, given $y(1) = 1$, $y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979$. 7

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