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Pages: 4

Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Regular and Supplementary Examination June 2023 (2019 Scheme)

Course Code: MAT256

Course Name: PROBABILITY AND STATISTICAL MODELLING

Max. Marks: 100 **Duration: 3 Hours**

(Statistical tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Given that $f(x) = \frac{k}{2^x}$ is a probability mass function of a random variable that can take on the values x=0,1,2,3 and 4 find (i) k and (ii)$P(X \leq 2)$. 3

2 Determine the binomial distribution for which mean 4 and variance is 3. 3

3 A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find P(X < 0) 3

4 The time in hours required to repair a machine is exponentially distributed with mean2. What is the probability that the time is (i) at most 1 hour (ii) at least 30 min? 3

5 A sample of 18 measurements of the diameter of a sphere gave a mean $\bar{X}$=5.24 inches and a standard deviation σ_s= 0.05 inches. Find a 99% confidence limits for the actual diameter. 3

6 Discuss the difference between Chi-square distribution and student's t-distribution 3

7 Define (i) Null hypothesis and Alternate hypothesis,(ii) Critical region. 3

8 Write down the format of the ANOVA table for one factor of classificaion. 3

9 Define the correlation coefficient. 3

10 Calculate the regression coefficients and obtain the lines of regression for the following data 3

X	1	3	10	16	26	36
Y	42	50	75	100	150	200

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Find th Mean and variance of Poisson distribution. 7

b) Six dice are thrown 729 times. How many times do you expect at least three dice to show 1 or 2? 7

12 a) In a given city 6% of all drivers get at least one parking ticket per year. Use the poisson approximation to the binomial distribution to determine the probabilities that among 80 drivers(randomly chosen in this city) 7

(i) 4 will get at least one parking ticket in any given year

(ii) At least 3 will get at least one parking ticket in any given year.

(iii) Anywhere from 3 to 6 inclusive, will get at least one parking ticket in any given year.

b) The joint probability distribution of X and Y is given by, $f(x,y) = \frac{1}{27}(2x+y)$: X= 0,1,2 and Y=0,1,2 7

(i) Find the marginal distribution of X and Y.

(ii) Are X and Y independent random variables.

Module -2

13 a) If X is uniformly distributed over $(\alpha, \alpha), \alpha < 0$. Find α so that (i) $P(X > 1) = \frac{1}{3}$ (ii) $P(|X| < 1) = P(|X| > 1)$ 7

b) The marks obtained in mathematics by 1000 students are normally distributed with mean 78% and standard deviation 11%. Determine 7

(i) How many students got marks above 90%

(ii) What was the highest mark obtained by the lowest 10% of students.

14 a) If X is normally distributed with mean 1 and variance 4, then 7

(i) find $P[-3 < X < 3]$ (ii) obtain k if $P[X \leq k] = 0.6$

b) A distribution with unknown mean μ has variance 1.5. Use Central Limit Theorem to find, how large a sample should be taken from the distribution in order that the probability that the sample mean will be with in the 0.5 of the population mean is 0.95. 7

Module -3

15 a) In a random selection of 64 of the 2400 intersections in a small city, the mean number of scooter accidents per year was 3.2 and the sample standard deviation was 0.8. 7

(i) Make an estimate of the standard deviation of the population from the sample standard deviation and work out the standard error of mean for this finite population.

(ii) If the desired confidence level is .90, what will be the upper and lower limits of the confidence interval for the mean number of accidents per intersection per year?

b) Suppose a certain hotel management is interested in determining the percentage of the hotel's guests who stay for more than 3 days. The reservation manager wants to be 95 per cent confident that the percentage has been estimated to be within ± 3% of the true value. What is the most conservative sample size needed for this problem? 7

16 a) A market research survey in which 54 consumers were contacted states that 54 percent of all consumers of a certain product were motivated by the product's advertising. Find the confidence limits for the proportion of consumers motivated by advertising in the population, given a confidence level equal to 0.95. 7

b) The foreman of ABC mining company has estimated the average quantity of iron ore extracted to be 36.8 tons per shift and the sample standard deviation to be 2.8 tons per shift, based upon a random selection of 4 shifts. Construct a 90% confidence interval around this estimate. 7

Module -4

17 a) It has previously been recorded that the average depth of ocean at a particular region is 67.4 fathoms. Is there a reason to believe this at 0.01 level of significance, if the reading at 40 random locations in that particular region showed a mean of 69.3 with standard deviation of 5.4 fathoms? 7

b) If 5 pieces of certain ribbon selected at random have mean breaking strength 169.5 pounds with standard deviation of 5.7, do they confirm to the specification mean breaking strength of 180 pounds. Use 0.01 level of significance. 7

18 a) If a random sample of 200 persons suffering with 'headache' 160 persons got cured by a drug, can we accept the claim of the manufacturer that his drug cures 90% of the suffers. Use 0.01 level of significance. 7

b) In order to determine whether there is significant difference in the durability of 3 makes of computers, samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are as follows. Carry out analysis of variance and give your conclusions. Use 0.01 level of significance. 7

A	5	6	8	9	7
B	8	10	11	12	4
C	7	3	5	4	1

Module -5

19 a) Suppose that in a certain chemical process the reaction time y(hr) is related to the temperature (°F) in the chamber in which the reaction takes place according to the simple linear regression model with equation $y=5.00-0.01x$ and $\sigma=0.075$ 7

What is the expected change in reaction time for a 1°F increase in temperature? For a 10°F increase in temperature?

b) Suppose that in a certain chemical process the reaction time y(hr) is related to the temperature (°F) in the chamber in which the reaction takes place according to the simple linear regression model with equation $y=5.00-0.01x$ and $\sigma=0.075$ 7

What is the probability that two independently observed reaction times for temperatures 1 degree apart are such that the time at the higher temperature exceeds the time at the lower temperature?

20 a) Suppose the variables x= commuting distance and y= commuting time are related according to the simple linear regression model with $\sigma=10$. If n=5 observations are made at the x values $x_1= 5, x_2= 10, x_3= 15, x_4= 20$, and $x_5=50$ calculate the standard deviations of the five corresponding residuals. 7

b) The ethylene content of lettuce seeds (y, in nL/g dry wt) was studied as a function of exposure time (x, in min) to an ethylene absorbent. 7

x	2	10	20	30	40	50	60	70	80	90	100
y	408	274	196	137	90	78	51	40	30	22	15

Estimate the parameters of $y = a + bx$ using the above data.

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