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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech Degree Supplementary Examination June 2023 (2019 Scheme)

Course Code: MAT 204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

(Statistical Tables are allowed)

PART A

(Answer all questions; each question carries 3 marks) Marks

1 A discrete random variable X has the following probability mass function 3

X	1	2	3	4	5	6
P(x)	0.30	0.25	0.15	0.05	0.10	0.15

Find the mean and variance.

2 The probability mass function of a discrete random variable is f(x)= kx, x=1,2,3 where k is a constant. Find the value of k and P ($X \leq 2$). 3

3 The error involved in making a certain measurement is a continuous random variable X with pdf f(x) = $\begin{cases} 0.09375(4-x^2) : -2 \leq x \leq 2 \\ 0 : otherwise \end{cases}$. 3

Compute (i) P (X > 0). (ii) P (-1< X< 1).

4. Derive the mean and variance of Uniform distribution. 3

5, A random process has the autocorrelation function $R_{XX}(\tau) = \frac{4\tau^2+6}{\tau^2+1}$. 3
Find the mean-square value, the mean value and the variance of the process.

6. Determine the autocorrelation function of the random process with the power spectral density given by $S_{XX}(w) = \begin{cases} S_0 & |w| < w_0 \\ 0 & otherwise \end{cases}$. 3

7. Find the root of the equation f(x) = $x^3 + x - 1$ using Regula Falsi Method in 3 stages correct to 4 decimal places. 3

8. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ dx using Trapezoidal rule taking 5 subintervals. 3

9. Using Euler's method, Solve for y at x= 0.05 from $\frac{dy}{dx} = x + y + xy$, 3

$y(0) = 1$ *taking* $h = 0.025$.

10, Using the Method of least squares, fit a straight line to the 4 points given below (-1.3, 0.103), (-0.1, 1.099), (0.2, 0.808) and (1.3, 1.897). 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11	a)	Derive the mean and variance of Poisson Distribution.	7
	b)	Suppose that 20% of all copies of a particular textbook fail a certain binding strength test. Let X denote the number among 15 randomly selected copies that fail the test. Then X has a binomial distribution with n =15 and p =0.2. Find (i) $P(X \leq 8)$ (ii) $P(X = 8)$ (iii) $P(X \geq 8)$ (iv) $P(4 \leq X \leq 7)$.	7
12	a)	Let X denote the number of creatures of a particular type captured in a trap during a given time period. Suppose X is a Poisson distribution with average traps will contain 4.5 creatures. Find the probability that (a) a trap contains exactly 5 creatures (b) a trap has at most five creatures.	7
	b)	The joint probability distribution of X and Y is given by $f(x, y) = C(2x + 3y)$; x =0,1,2 and y = 1,2,3. Find (i) the value of C. (ii) the marginal distributions of X and Y. (ii) Are X and Y independent random variables.	7

Module -2

13	a)	Data collected at Toronto Pearson International Airport suggests that an exponential distribution with mean value 2.725 hours is a good model for rainfall duration. (i) What is the probability that the duration of a particular rainfall event at this location is at least 2 hours? (ii) At most 3 hours? (iii) Between 2 and 3 hours?	7
	b)	The joint pdf of two random variables X and Y is given by $F(x, y) = \begin{cases} kxy & 0 < x < 4, 1 < y < 5 \\ 0 & otherwise \end{cases}$. Find (i) k (ii) $P(X \geq 3, Y \leq 4)$ (iii) Check whether X and Y are independent?	7
14	a)	Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with mean 8.8 and standard deviation 2.8. (i) What is the probability that the diameter of a randomly selected tree will be at least 10 in.? (ii) What is the probability that the diameter of a randomly selected tree will exceed 20 in.? (iii) What is the probability that the diameter of a randomly selected tree will be between 5 in and 10 in.?	7
	b)	The life time of a certain brand of tube light may be considered as a random variable with mean 1200 hours and standard deviation 250 hours. Using Central limit theorem, find the probability that the average life time of 60 lights exceeds 1250 hours.	7

Module -3

15	a)	Assume that X(t) is a random process defined as follows: $X(t) = A\cos(2\pi t + \emptyset)$ where A is a zero-mean normal random variable with variance $\sigma^2_A = 2$ and $\emptyset$ is uniformly distributed random variable over the interval $-\pi \leq \varphi \leq \pi$. A and φ are statistically independent. Let the random variable Y be defined as $Y = \int_0^1 X(t)dt$. Determine (i) the mean of Y. (ii) the variance of Y.	7
	b)	A random process X(t) is defined by $X(t) = a\cos(wt+\theta)$ where a and w are constants and θ is uniformly distributed in $[0,2\pi]$. Show that X(t) is WSS.	7

16 a) Show that the sum of two independent Poisson processes is also a Poisson process. 7

b) Cell-phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute. (i) What is the probability that more than three calls arrive in an interval of length 20 seconds? (ii) What is the probability that more than 2 calls arrive in each of two consecutive intervals of length 30 seconds? 7

Module -4

17 a) Find the positive solution of 2 sinx = x starting with $x_0 = 2$ using Newton Raphson's method up to 4 decimal places 7

b) If f(x) = coshx, using these 4 values (x_0 , f_0) = (0.5, 1.127626) (x_1 ,f_1) = (0.6, 1.185465) (x_2 ,f_2) = (0.7, 1.255169) (x_3 ,f_3) = (0.8, 1.337135). Find f (0.56) using Newton's Forward Difference formula. 7

18 a) Find the Newton's Divided difference polynomial for the data in the table given below. Evaluate f (2.5) 7

x	-3	-1	0	3	5
f(x)	-30	-22	-12	330	3458

b) Evaluate $\int_0^2 x\, e^x\, dx$ using Simpson's rule with 8 sub intervals. Compare it with its actual value. 7

Module -5

19 a) Solve the system of linear equations using Gauss Seidel method 10x -5y- 2z =3 ,4x-10y + 3z =-3 and x + 6y+ 10z = -3 up to 4 decimal places. 7

b) Using Runge Kutta Method of second order, solve the initial value problem by finding the value of y (0.05) 7

$$\frac{dy}{dx} = x + y + xy \text{ where } y(0) = 1 \text{ taking } h = 0.025$$

20 a) Using Runge Kutta Method of fourth order the solution of $\frac{dy}{dx} = \frac{y-x}{y+x}$ *in the interval* (0,1) *gives* the following set of values y(0) = 1 , y(0.1) = 1.0911 y(0.2)=1.1678 y(0.3)=1.2335 . Find y (0.4) using Adam Moulton's predictor corrector method. 7

b) Use Runge-Kutta method of fourth order to find y (0.1) given the initial value problem $\frac{dy}{dx} = \frac{xy}{1+x^2}$ *where* y (0) = 1 and take step-size h = 0.1. 7

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