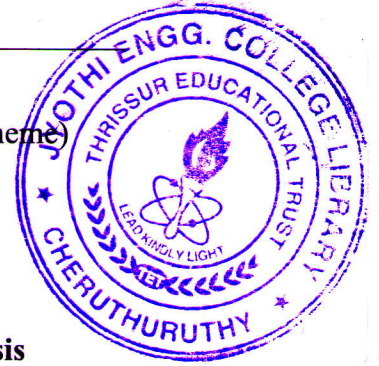


Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
Third Semester B.Tech Degree Examination December 2021 (2019 scheme)



**Course Code: MAT201**

**Course Name: Partial Differential equations and Complex analysis**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions. Each question carries 3 marks*

- |    |  | Marks |
|----|--|-------|
| 1  | Find the partial differential equation by eliminating arbitrary functions $f$ and $g$ from $z = f(x) + g(y)$     | (3)   |
| 2  | Solve $\frac{\partial^2 z}{\partial x^2} = xy$   | (3)   |
| 3  | Write the three possible solutions of one dimensional wave equation.   | (3)   |
| 4  | Write any two assumptions used in the derivation of one dimensional heat equation.                               | (3)   |
| 5  | Test the continuity at $z = 0$ of $f(z) = \begin{cases} \frac{Im(z)}{ z }, & z \neq 0 \\ 0, & z = 0 \end{cases}$ | (3)   |
| 6  | Check whether $f(z) = \bar{z}$ is an analytic function?  | (3)   |
| 7  | Evaluate $\oint_c \frac{e^z}{z-5} dz$ , where $c$ is the circle $ z =4$  | (3)   |
| 8  | Find the Taylor series expansion of $e^z$ about $z = \pi$ .  | (3)   |
| 9  | Give example of (a) removable singularity (b) pole (c) essential singularity                                     | (3)   |
| 10 | Find the Laurent series expansions of $\frac{1}{z(z-1)}$ about $z = 0$   | (3)   |

**PART B**

*Answer any one full question from each module. Each question carries 14 marks*

**Module 1**

- |    |   |   |
|----|---|---|
| 11 | (a) Solve $y^2 p - xyq = xz$  | 7 |
|    | (b) Solve by the method of separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ , $u(x, 0) = 4e^{-3x}$ | 7 |
| 12 | (a) Find the complete integral of $px + qy = pq$ using Charpit's method.  | 7 |
|    | (b) Form the partial differential equation corresponding to family of spheres with center on $z$ -axis and radius $a$ .                       | 7 |

Module 2

- 13 (a) Solve the boundary value problem described by  $u_{tt}-c^2u_{xx}=0, 0 \leq x \leq \ell, t \geq 0$  7  
 $u(0, t) = u(\ell, t) = 0, t \geq 0, u(x, 0) = 10 \sin\left(\frac{\pi x}{\ell}\right), \frac{\partial u}{\partial t}(x, 0) = 0$
- (b) Find the temperature  $u(x, t)$  in a homogenous bar heat conducting material of length  $l$  whose ends kept at  $0^\circ\text{C}$  and whose initial temperature is given by  $u(x, 0) = lx - x^2$ . 7
- 14 (a) Derive one dimensional wave equation. 7
- (b) The ends A and B of a rod 10 cm in length are kept at temperatures  $0^\circ\text{C}$  and  $100^\circ\text{C}$  until the steady state condition prevails. If B is Suddenly reduced to  $0^\circ\text{C}$  and kept so . Find the temperature distribution in the rod at time  $t$ . 7

Module 3

- 15 (a) Show that an analytic function  $f(z) = u + iv$  is constant if its modulus is constant. 7
- (b) Find the image of  $1 \leq |z| \leq 2, \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$  under the mapping  $w = z^2$  7
- 16 (a) Verify whether  $u = x^3 - 3xy^2$  is harmonic and find its conjugate harmonic function  $v$ . 7
- (b) Find the image of the region between real axis and a line parallel to real axis at  $y = \frac{\pi}{2}$  under the mapping  $W = e^z$  7

Module 4

- 17 (a) Evaluate  $\int_C |z|^2 dz$  where  $C$  is the circle  $|z| = 2$ . 7
- (b) Evaluate  $\int_C \frac{z^2+2}{(z-3)^2} dz$  where  $C$  is the circle  $|z| = 4$  using Cauchy's integral formula 7
- 18 (a) Evaluate  $\oint_C \frac{e^z}{(z-1)(z-4)} dz$ , where  $c$  is  $|z| = 2$  using Cauchy's integral formula 7
- (b) Evaluate  $\int \frac{3z^2+7z}{z+1} dz$  over (a)  $|z| = 1.5$  (b)  $|z + i| = 1$  7

Module 5

- 19 (a) Find the Laurent series expansion for  $f(z) = \frac{1}{(z-1)(z-2)}$  valid in (a)  $1 < |z| < 2$  (b)  $|z| > 2$  7

- (b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$
- 20 (a) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2+2}{(x^2+1)(x^2+4)} dx$  7
- (b) Using residue theorem evaluate  $\oint_c \frac{z+1}{z^4-2z^3} dz$ , where  $c$  is  $|z| = \frac{1}{2}$  7

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