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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech (Minor) Degree Examination June 2022 (2020 Admission)

Course Code: CST284

Course Name: Mathematics for Machine Learning

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Let V = { x, 1/ 2 x : x real number} with standard operations. Is it a vector space? Justify your answer 3

2 Let $x_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, x_1 = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, x_3 = \begin{bmatrix}1\\2\\1\\2\end{bmatrix}$, and $x_4 = \begin{bmatrix}3\\5\\5\\7\end{bmatrix}$. Is $\{x_1, x_2, x_3, x_4\}$ linear dependent or linearly independent? 3

3 Is the following matrix diagonalizable? Explain 3

$$A = \begin{bmatrix}1 & 2 & 3\\0 & 5 & 8\\0 & 0 & 13\end{bmatrix}$$

4 $A = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$ .Find the number of distinct eigenvalues of A without calculating determinant 3

5 Find all critical points of *f(x)*= sin *x* + cos *x* on [0,2π]. 3

6 Find the third -degree Taylor polynomial for $f(x) = x^3 + 7x^2 - 5x + 1$ about x=0. 3

7 The length of time, in minutes, that a customer queues in a Post Office is a random variable, T, with probability density function 3

$$f(t) = \begin{cases} c(81 - t^2) & 0 \le t \le 9 \\ 0 & otherwise \end{cases}$$ where c is a constant

Show that the value of c is $\frac{1}{486}$

8 Two dice are rolled. Consider the events A = {sum of two dice equals 3}, B = {sum of two dice equals 7}, and C = {at least one of the dice shows a 1}. What is P (A | C)? 3

9 A linear programming problem has objective function P = 3x + 2y and the 3

following linear inequality constraints.

$$x - y \leq 0, \quad x + y \leq 3, \quad x \geq 0, \quad y \geq 0$$

How many slack variables are needed for the simplex algorithm?

10 Consider the function $2x^2 + 4y^2$ on the set $x^2 + y^2 = 1$. Use Lagrange multipliers to find the global minimum and maximum of this function. What do the second order criteria say at (1, 0)? 3

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Find all solutions to the system of equations 7

$$2w + 3x + 4y + 5z = 1$$
$$4w + 3x + 8y + 5z = 2$$
$$6w + 3x + 8y + 5z = 1$$

b) Use matrix inverse methods to solve each of the following systems: 7

$x_1 - x_2 + x_3 = 3$

$2x_2 - x_3 = 1$

$2x_1 + 3x_2 = 4$

12 a) Find Ker T,where $T: E^3 \rightarrow E^2$ is defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 - x_3)$ 4

b) Show that the following transformation are linear 10

$(i) T: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ defined by $T((z_1, z_2)) = (z_1 + z_2, z_1 - 2z_2)$

(ii) $T: E^3 \rightarrow E^3$ defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1$

Module -2

13 a) Let $A = \begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}$ (a) Is A orthogonally diagonalizable? If so, orthogonally diagonalize it 4

b) Find the SVD of $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ 10

14 a) Let S be the subspace of $\mathbb{R}^4$ spanned by the vectors 7

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Find a Gram-Schmidt orthonormal basis of S.

b) Find the orthogonal projection vector v of v_2 onto v_1, given 7

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Module -3

15 a) Calculate all four second partial derivatives for the function f(x,y)=sin(3x−2y)+cos(x+4y). 8

b) How to determine whether this function is differentiable at a point? 6

$$f(x) = \begin{cases} \dfrac{x}{1+x} & x \geq 0 \\ x^2 & x < 0 \end{cases}$$

16 a) Find the Taylor series for e^{-x^2} centered at 0 7

b) Use the first two non-zero terms of an appropriate series to give an approximation of 7

$$\int_0^1 \sin x^2 \, dx$$

Module -4

17 a) Assume A and B are independent events with P(A) = 0.2 and P(B) = 0.3. Let C be the event that neither A nor B occurs, let D be the event that exactly one of A or B occurs. 8

Find (i) P(C) (ii) P(D) (iii) P(A|D) (iv) Are C and D independent

b) Suppose A, B, and C are mutually independent events with probabilities P(A) = 0.5, P(B) = 0.8, and P(C) = 0.3. Find the probability that at least one of these events occurs 6

18 a) An insurance policy is written to cover a loss, X, where X has uniform distribution on [0, 1000]. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible? 8

b) Suppose that the number of customers visiting a fast food restaurant in a given day is N~Poisson(λ). Assume that each customer purchases a drink with probability p, independently from other customers, and independently from the value of N. Let X be the number of customers who purchase drinks. Let Y be the number of customers that do not purchase drinks; so X+Y=N. 6

(i)Find the marginal PMFs of X and Y.

(ii)Find the joint PMF of X and Y.

(iii)Are X and Y independent?

Module -5

19 a) Maximize 8

$$f(x) = 2x_1 + 3x_2 - x_1^2 - x_2^2$$

Subject to

$$x_1 + x_2 \leq 2$$

$$2x_1 + x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

b) Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line y =3x-3 6

20 a) A furniture company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hours in the painting department. Each table takes 4 hours of carpentry and 2 hours in the painting department. Each chair requires 3 hours of carpentry and 1 hour in the painting department. During the current production period, 240 hours of carpentry time are available and 100 hours in painting is available. Each table sold yields a profit of E7; each chair produced is sold for a E5 profit. Find the best combination of tables and chairs to manufacture in order to reach the maximum profit 10

b) Write a short note on linear programming 4

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